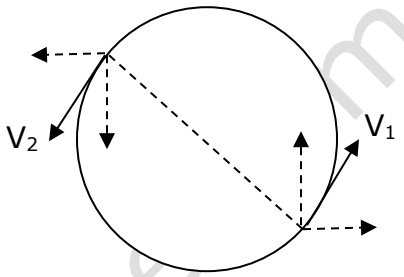
Q- A cat rides a merry-to-round turning in circular motion with uniform speed. At time t_1 = 2.00s, the cat's velocity is V_1=3.0 m/s i + 4.0 m/s j, measured on a horizontal x-y coordinate system. At t_2 = 5.00s, its velocity is V_2= (-3.0 m/s) i + (-4.0 m/s) j. What are the

(a) magnitude of the cat's centripetal acceleration

The centripetal acceleration of the particle moving on a circular path of radius R with speed v is given by

$$a = \frac{v^2}{R} = \omega^2 R = v\omega \quad \text{[as v = } \omega R\text{]}$$



The speed of the cat at any instant will be

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{3.0^2 + 4.0_y^2} = \sqrt{9+16} = 5 \text{ m/s}$$

As the direction of the x component and y components of the velocities are just reversed in time $t_2 - t_1$ = 5.00 - 2.00 = 3.00 s (negative signs) we can say that in this time the cat makes half rotation and hence the time required for full rotation i.e. the time-period is 6.00 s. Thus, the angular velocity of the cat can be given by

$$\omega = \frac{2\pi}{T} = \frac{2*3.14}{6.00} = 1.047 \quad \text{rad/s}$$

Hence the centripetal acceleration of the cat will be

$$a = v*\omega = 5.0*1.047 = 5.235 \text{ m/s}^2$$

(b) the cat's average acceleration during the time interval t_2-t_1?

The average acceleration of the cat can be given by

$$\langle a \rangle = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\left(-3.0*\hat{i} - 4\hat{j}\right) - \left(3.0*\hat{i} + 4.0\hat{j}\right)}{5.00 - 2.00} = \frac{-6\hat{i} - 8\hat{j}}{3} = -2\hat{i} - \frac{8}{3}\hat{j} \text{ m/s}^2$$

Hence the magnitude of the average acceleration is

$$a = \sqrt{2^2 + \left(\frac{8}{3}\right)^2} = 3.33 \text{ m/s}^2$$

and its direction makes angle

$$\theta = \tan^{-1}\left(\frac{-8/3}{-2}\right) = \tan^{-1}(1.33) = 53^0 \text{ (with negative x axis, third quadrant)}$$