Q- An apparatus similar to the one used in lab uses an oscillating motor at one end to vibrate a long rope with frequency f = 35 Hz and amplitude A = 0.25 m. The rope is held at constant tension by hanging a mass on the other end. The length of the rope is adjusted to form loops. The rope has mass density μ= 0.020 kg/m, and tension T = 17.27 N. Assume that at t = 0 the end of the rope at x = 0 has zero y-displacement. What is the y-displacement of the piece of rope at x_1 = 0.40 m when t = 0?

This is the experiment based on standing waves.

The wave created by the oscillator in the rope is reflected back from the fixed point (pulley) and hence there are two waves in the rope in opposite directions producing standing waves.

As the oscillator is at x= 0, this point of the rope will be an anti-node and the displacement y is zero here at t = 0. Hence the equation of the standing wave can be written as

$$y(x,t) = (2A\cos kx)\sin \omega t \qquad \text{------------------ (1)}$$

The velocity of a wave on a stretched string is given by

$$c = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{17.27}{0.020}} = 29.385 \text{ m/s}$$

Hence the wavelength of the wave produced on the string will be

$$\lambda = \frac{c}{f} = \frac{29.385}{35} = 0.84\,\text{m}$$

Thus, the wave number

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.84} = 2.38\pi$$

Substituting the values in equation (1) the displacement of the point x = 0.40 m at t = 0 is given by

$$y_1 = (0.25m)\cos(2.38\pi * 0.40)\sin(w * 0) = 0$$

Or $\quad y_1 = 0$

This can be understood directly as all the particles of the rope comes to equilibrium at the same instant twice in one oscillation, any particle will be at equilibrium when the source oscillator is at equilibrium hence at t = 0 at all points of the rope y displacement will be zero.