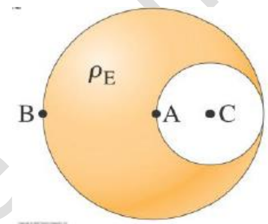
Q. A sphere of radius r_0 carries a volume charge density ρ_E. A spherical cavity of radius $r_0/2$ is then scooped out and left empty, as shown.

The charge on the sphere is distributed on volume means that the sphere is nonconducting and so by making cavity the remaining portion of sphere will have a charge density ρ_E, while the cavity is having no charge.

This system may be considered as superposition of two charge distributions. One the sphere of radius r_0 of density ρ_E and the other the sphere of radius $r_0/2$ (cavity) with charge density - ρ_E. The positive and negative densities make the cavity charge less. Thus, the field at any place can be considered as superposition of two fields due to each charge distribution.



(a) What is the magnitude and direction of the electric field at point A?

A is the center of the larger sphere thus the field at A due to the whole charged sphere of radius r_o with density ρ_E is zero.

$$E_1 = 0$$

A is at the surface of the small sphere or radius ro/2 with density – ρ_E thus the field at A due to negatively charged small sphere will be

$$E_2 = \frac{Q'}{4\pi\epsilon_0 \left(\frac{r_0}{2}\right)^2} = \frac{\frac{4}{3}\pi\left(\frac{r_0}{2}\right)^3 * (-\rho_E)}{4\pi\epsilon_0 \left(\frac{r_0}{2}\right)^2} = -\frac{\rho_E\, r_0}{6\epsilon_0}$$

The negative sign shows that because of the negative charges the direction of field is towards the center of the cavity.

Thus the magnitude of the total field at A is $\frac{\rho_E\, r_0}{6\epsilon_0}$ and its direction is A to C

(b) What is the direction and magnitude of the electric field at point B? Points A and C are at the centers of the respective spheres.

At point B as it is at the surface of the sphere, the field due to positively charged sphere of radius r_0 is same as the whole charge is at its center and is given by

$$E_1 = \frac{Q}{4\pi\epsilon_0\, r_0^2} = \frac{\frac{4}{3}\pi r_0^3 * \rho_E}{4\pi\epsilon_0\, r_0^2} = \frac{\rho_E\, r_0}{3\epsilon_0}$$

This field will be radially outwards.

As point B is at a distance 3ro/2 from the center C of cavity sphere of radius $r_0/2$, the field due to this negatively charged sphere at B is given by

$$E_2 = \frac{Q'}{4\pi\epsilon_0 \left(\frac{r_0}{2}\right)^2} = \frac{\frac{4}{3}\pi\left(\frac{r_0}{2}\right)^3 * (-\rho_E)}{4\pi\epsilon_0 \left(\frac{3\, r_0}{2}\right)^2} = -\frac{\rho_E\, r_0}{54\,\epsilon_0}$$

Thus, the resultant field at B is given by

$$E = E_1 + E_2 = \frac{\rho_E\, r_0}{3\epsilon_0} - \frac{\rho_E\, r_0}{54\,\epsilon_0} = \frac{17\,\rho_E\, r_0}{54\,\epsilon_0}$$

Thus the magnitude of the field at B will be $\frac{17\,\rho_E\, r_0}{54\,\epsilon_0}$ and it will be radially outward means from A to B.