Q- A spherical tank contains 1 mole of helium gas at 10bar and 300K. The gas is slowly released to the atmosphere in such a way the remaining gas is maintained at constant total energy U. Heating or cooling may be used to keep U constant during the venting. Helium behaves as an ideal gas with a constant Cv =12.6J/mol.K. The base state is that the specific energy, U, is zero at 300 K. When one-half of the helium has been vented, what is the temperature and pressure of the residual helium?

Let initial pressure is P_0 = 10 bar, volume of the tank is V_0 and the initial temperature of the gas is T_0 = 300 K. Let at some time pressure of the in the tank gas is P, the number of moles is n its temperature is T.

Let a small amount of the gas (dn moles) is venting to atmosphere in this situation and because of this the pressure reduces by dP.

The internal energy of n moles of ideal gas is given by

$$U = nC_v T \qquad \text{------------------------- (1)}$$

And the internal energy after dn moles vented will be

$$U_2 = (n - dn)C_v(T + dT)$$

Hence the change in the internal energy of the gas is given by

$$\Delta U = U_2 - U_1 = C_v(-T * dn + n * dT) \qquad \text{------------------------- (2)}$$

As this dn mole of the gas is pushed out the internal energy of the gas is reducing in two ways

1. due to work done by the gas in side to push dn mole out and
2. the energy carried away by the venting gas.

Hence the loss of energy is given by this way as well

$$\Delta U = -(dW + dnRT)$$

Or $$\Delta U = -(P * dV + dnRT)$$

Here dV is the volume of the gas vented given by dV = dn*RT/P. hence we get

$$\Delta U = -\left(P * \frac{dn * RT}{P} + dnRT\right) = -2 * dnRT \qquad \text{----------------- (3)}$$

As the lost energy will be same with both ways of calculation, from equations 2 and 3 we get

$$\Delta U = C_v(-T * dn + n * dT) == -2 * dnRT$$

$$C_v(-T * dn + n * dT) = -2 * dnRT$$

or $$nC_v dT = (C_v - 2R)T * dn$$

Gives $$\frac{dT}{T} = \left(1 - \frac{2R}{C_v}\right) * \frac{dn}{n}$$

Integrating the equation, we have

$$\int_{T0}^{T} \frac{dT}{T} = \left(1 - \frac{2R}{C_v}\right) * \int_{n0}^{n} \frac{dn}{n}$$

Gives $$\ln\left(\frac{T}{T_0}\right) = \left(1 - \frac{2R}{C_v}\right)\ln\left(\frac{n}{n_0}\right)$$

Or $$\frac{T}{T_0} = \left(\frac{n}{n_0}\right)^{\left(1-\frac{2R}{C_v}\right)}$$ ------------ (4)

As finally half of the gas remains in the tank the number of moles n = n_0/2, and the temperature at this time will be given by

$$\frac{T}{T_0} = \left(\frac{1}{2}\right)^{\left(1-\frac{2R}{C_v}\right)}$$

Or $$T = 2^{\left(\frac{2R}{C_v}-1\right)}T_0 = 2^{\left(\frac{2*8.31}{12.6}-1\right)}T_0 = 2^{\left(\frac{2*8.31}{12.6}-1\right)}*300 = 2^{0.319}*300 = 1.2475*300 = 374.24 \text{ K}$$

Now initial number of moles $n_0 = \frac{P_0V_0}{RT_0}$ and n = $\frac{PV_0}{RT}$ we have

$$\frac{n}{n_0} = \frac{P}{T}*\frac{T_0}{P_0}$$ ---------------- (5)

Substituting in equation 4 we get the relation between P and T as

$$\frac{T}{T_0} = \left(\frac{P}{T}*\frac{T_0}{P_0}\right)^{\left(1-\frac{2R}{C_v}\right)}$$

Gives $$\frac{T}{T_0} = \left(\frac{P}{T}*\frac{T_0}{P_0}\right)^{\left(1-\frac{2R}{C_v}\right)}$$

Or $$\left(\frac{P}{P_0}\right)^{\left(1-\frac{2R}{C_v}\right)} = \left(\frac{T}{T_0}\right)^{\left(2-\frac{2R}{C_v}\right)}$$ ---------------- (6)

Or $$\left(\frac{P}{P_0}\right)^{-0.319} = \left(\frac{T}{T_0}\right)^{0.681}$$

Or $$\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{0.681}{-0.319}} = \left(\frac{374.24}{300}\right)^{-2.135} = (1.2475)^{-2.135} = 0.624$$

Or P = 0.624 P_0 = **6.24 bar**

Substituting the values when half gas is vented we have

$$\left(\frac{4}{10}\right)^{3-2*1.66} = \left(\frac{300}{T}\right)^{2(1.66-1)}$$

Or $$\left(\frac{4}{10}\right)^{-0.32} = \left(\frac{300}{T}\right)^{1.32}$$

Or $$\frac{T}{300} = \left(\frac{4}{10}\right)^{\frac{0.32}{1.32}} = 0.8$$

Or T = 300*0.8 **= 240 K**