Q- A block slides 6 feet down the incline of an escalator and then it slides for 2 feet along the level portion until if flies off. If the level portion of the escalator is 3 feet above the floor, determine how far from the base of the escalator the block will strike the floor. Assume that the coefficient of kinetic friction between the block and the escalator is 0.15 and that the angle of the incline is 30 degrees.

Solution:

The block accelerates on the incline and then decelerates on the plane surface.

On the incline, the forces acting on the block are its weight mg, the normal reaction of the inclined and the friction. The diagram shows the forces. As we know that the block is moving along the incline the net force normal to the plane must be zero.

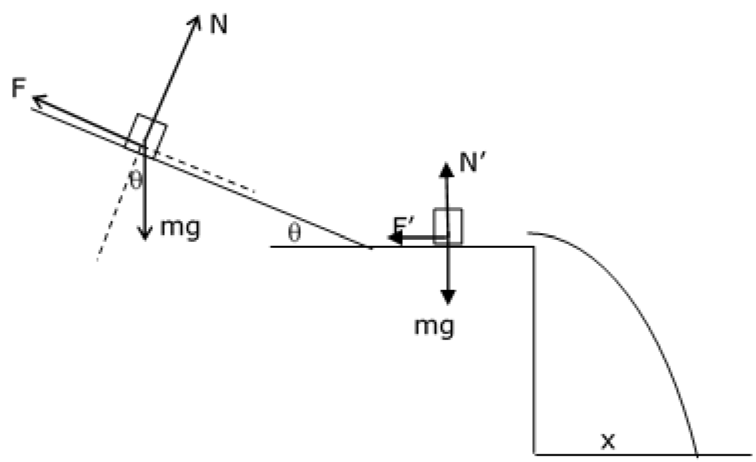
Resolving the weight of the block along and normal to the inline we have

Component of the weight along the incline is mg*cos θ and in the direction normal to the incline mg*sin θ.

Now the block is not moving normal to the incline hence the normal forces are balanced hence

$$N - mg\cos\theta = 0$$

gives $N = mg\cos\theta$



Friction force between the block and the incline will be

$$F = \mu*N = \mu*mg*\cos\theta$$

Hence writing equation of motion of the block along the incline in downward direction we have. (Acceleration due to gravity $g = 32\ f/s^2$)

$$mg*\sin\theta - \mu*mg*\cos\theta = ma$$

Gives $a = g(\sin\theta - \mu*\cos\theta) = 32(\sin 30^0 - 0.15*\cos 30^0) = 11.84\ ft/s^2$

So, the acceleration of the block down the incline is $11.84\ ft/s^2$.

Initial velocity of the block u = 0 and the distance moved on the incline is 6 ft hence the final velocity v of the block at the end of the incline is given by using third equation of motion as

$$\{v^2 = u^2 + 2*a*s\}$$

or $v^2 = 0 + 2*11.84*6.0 = 142.1$

or $v = 11.92$ ft/s.

Now on the plane surface the weight of the block is balanced by the normal reaction and hence $N' = mg$

The friction force in the direction opposite to the direction of motion is $F' = \mu N'$

And equation of motion of the block will be

$$-\mu * mg = m*a'$$

(Negative sign is due to the direction of friction is opposite to the direction of motion)

Hence acceleration of the block on the plane surface will be

$$a' = -\mu g = -0.15*32 = -4.8 \text{ f/s}^2.$$

Velocity v' of the block after moving s' = 2 ft before falling is given by

$$\{v^2 = u^2 + 2*a*s\}$$

or $v'^2 = 11.92^2 + 2(-4.8)*2.0 = 122.89$

or $v = 11.09$ ft/s

The block falls form the plane surface with zero vertical velocity (only horizontal velocity of 11.09 ft/s) under gravity and hence time taken to reach the floor is given by

$$h = ut + ½ gt^2$$

Or $3 = 0 + ½ * 32*t^2$

Gives, $t = 0.433$ s

In this time, horizontal distance covered with constant horizontal velocity of 11.09 ft/s is

x = 11.09*0.433 =4.80 ft.

Hence the block will land 4.8 feet from the base of the accelerator.