Q- A body is projected vertically with a velocity of 6 km/s. Find the maximum height reached by projectile.

The velocity of the projectile 6 km/s is large enough to send it at such height for which acceleration due to gravity cannot be considered constant and thus we must consider the variation in acceleration due to gravity as well.

The potential energy U of a body of mass m at distance r from the center of earth above the surface of the earth is given by

$$U = -\frac{GMm}{r}$$ (Here G is gravitation constant and M is the mass of earth.)

Hence the potential energy of the projectile at the surface of earth is given by

$$U_1 = -\frac{GMm}{R}$$ (Here R is the radius of the earth)

The potential energy of the projectile at height h from the surface of earth is given by

$$U_2 = -\frac{GMm}{R+h}$$

Hence the increase in potential energy of the projectile when it is raised to height h will be

$$U_2 - U_1 = \left(-\frac{GMm}{R+h}\right) - \left(-\frac{GMm}{R}\right) = GMm\left(-\frac{1}{R+h} + \frac{1}{R}\right) = \frac{GMmh}{(R+h)R} \qquad \text{---- (1)}$$

Now the initial Kinetic energy of the projectile is given by

KE = (1/2) mv^2

As at the maximum height reached the velocity of the projectile becomes zero hence the final kinetic energy of the projectile will be zero hence according law of conservation of total energy we can get

Gain in potential energy = loss in kinetic energy

Or $$\frac{GMmh}{(R+h)R} = \frac{1}{2}mv^2$$

Or $$\frac{2GMh}{(R+h)R} = v^2$$

Now as at the surface of earth acceleration due to gravity is given by

$$g = \frac{GM}{R^2}$$

From hear substituting the value of GM in the equation above we get

$$\frac{2gR}{v^2} = \frac{R+h}{h}$$

Or $$\frac{2gR}{v^2} - 1 = \frac{R}{h}$$

Or $$h = \frac{Rv^2}{(2gR - v^2)} = \frac{6.37*10^6*(6000)^2}{2*9.8*6.37*10^6 - (6000)^2} = 2.581*10^6 \text{ m}$$

Or h = 2581 km.