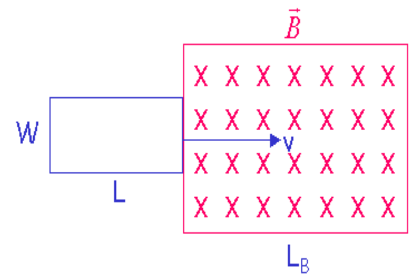
Q- At t=0, a rectangular coil of resistance R = 2 ohms and dimensions w = 3 cm and L = 8 cm enters a region of constant magnetic field B = 1.6 T directed into the screen as shown. The length of the region containing the magnetic field is L_B = 15 cm. The coil is observed to move at constant velocity v = 5 cm/s. What is the force required at time t = 0.8 sec to maintain this velocity?

According to Faraday's laws of electromagnetic induction the induced EMF in a closed loop is given by

$$\varepsilon = -\frac{d\phi_B}{dt} \qquad \text{------------------------ (1)}$$

Here the sign is according to the Lenz law, shows that the EMF in the loop will be in such a way that it opposes the cause due to which it is produced.



Now as the flux through a loop is given by

$$\phi_B = \vec{B} \bullet \vec{A} = BA\cos\theta$$

And here the area vector $\vec{A}$ is parallel to the magnetic field vector $\vec{B}$, θ = 0 or cos θ = 1 hence the flux through the coil at any time is given by

$$\phi_B = BA$$

Where A is the area of the coil in the magnetic field

As the magnetic field strength B is constant and the area in the field is varying, according to the Faraday's law we have

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} \qquad \text{------------------------ (2)}$$

The distance moved by the coil in time t is given by

x = v*t

[In time t = 0.8 s the distance covered by the coil is

x (0.8s) =5.0*0.8 = 4.0 m

And hence the coil is still going in to the field and the flux through the coil is increasing.
The area of the coil in the field at time t will be

A = w*x = w*v*t

And hence the induced EMF in the coil is given by equation (2) as

$$\varepsilon = -B\frac{d(wvt)}{dt} = -Bwv$$

[As B, w and v all constant the induced EMF in the coil will be constant]

Thus the ***magnitude*** of current in the loop at time t (<8/5 = 1.6s) is given by

$$I = \frac{\varepsilon}{R} = \frac{Bwv}{R}$$

Now as the force F acting on a wire of length w carrying current I perpendicular to a magnetic field B is given by F = B*I*w, the force on the wire is given by

$$F = B*\left(\frac{Bwv}{R}\right)*w = \frac{B^2w^2v}{R} = \frac{1.6^2 * 0.03^2 * 0.05}{2} = 5.76*10^{-5}\ \text{N}$$

Hence to maintain velocity equal and opposite force to be applied on the coil.

F(0.8 sec) = 5.76*10^{-5} N