Q- A ball is rolling up an incline with a velocity of 7 m/s. What will be its velocity when it reaches a vertical height h = 2 m above its initial position?

The linear velocity of the ball is v = 7 m/s.

When a ball or a wheel is rolling without slipping, the point of contact with the surface is momentarily at rest, means the velocities at this point at that moment, due to translation and rotation are equal and opposite and this is possible only if the relation between its angular velocity w and the linear velocity is given by

$\omega = v/R$

As when the ball is rolling its center of mass is translating with velocity v as well as it rotates about the axis passing through its center with angular velocity $\omega = v/R$. It possesses kinetic energy corresponding to rotation as well as translation and hence its total kinetic energy is given by

Total KE = Translational KE + Rotational KE

Or $$KE = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$

Now as the ball rolls over the incline its potential energy increases and kinetic energy decreases.

Using the law of conservation of mechanical energy, we can write

Loss in kinetic energy = Gain in potential energy

Or $$\frac{7}{10}M*v^2 - \frac{7}{10}M*v_f^{\,2} = Mgh$$

Or $$v^2 - v_f^{\,2} = \frac{10}{7}gh$$

Or $$v_f^{\,2} = v^2 - \frac{10}{7}gh$$

Or $$v_f^{\,2} = 7^2 - \frac{10}{7}*9.8*2 = 49 - 28 = 21$$

Gives $v_f = \sqrt{21} = 4.58$ m/s