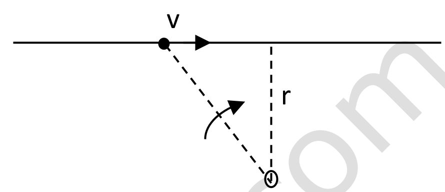
Q- A disk of mass M = 400 g and radius R = 30 cm is free to rotate about its horizontal axis. A lump of clay of mass m = 100 g falls from height h = 2m and sticks to the disk at the farthest point from the center. Find the angular velocity of the disk after collision.

As a particle moving on a straight line with velocity v is seen by a stationary observer not in the line of motion then he must rotate the line of sight with the particle and hence we can say that the particle is rotating about that stationary point. Thus, the particle of mass m moving with speed v on a straight line, at distance r possess an angular momentum and this is given by the product of linear momentum of the particle and the perpendicular distance of line of motion.



That is why the angular momentum is called the moment of momentum.

Now the clay, during the fall through gravity by 2 m will acquire a velocity v, just before collision is given by the law of conservation of energy as

Gain in KE = loss in PE

Or $\frac{1}{2}mv^2 = mgh$

Or $v = \sqrt{2gh} = \sqrt{2*9.8*2} = 6.261$ m/s

Hence its momentum just before collision will be

P = m*v = 0.400*6.261 = 2.5 kg.m/s

And its angular momentum about the axis of rotation just before collision is given by

L = P*R = mv*R = 2.5*0.3 = 0.75 kg.m^2/s^2

Now as there is no external torque about axis of rotation just at the time of collision, considering the disk and the clay as a system, its angular momentum remains conserved and if after collision the combined mass rotates about the axis with angular velocity w just after collision we have

Final angular momentum = initial angular momentum

Or $I*\omega$ = o + L = mv*R

As we know that the moment of inertia of a disc is $MR^2/2$ and that of clay is mR^2 about the axis of rotation we have

$$\left(\frac{1}{2}MR^2 + mR^2\right)\omega = mvR$$

Or $$\omega = \frac{2mvR}{\left(MR^2 + 2mR^2\right)} = \frac{2mv}{MR + 2mR} = \frac{2*0.400*6.261}{(2+2*0.400)*0.3} = 5.96 \text{ rad/s}$$