Q - The position vector of a particle moving on the rim of a rolling wheel in the xy plane is given by r = (10t + 5 cos 2t)i + (5 - 5sin 2t)j, where r is in meters and t is in seconds. (a) What are the velocity and speed of the particle? (b) Sketch the path that the particle traces out as the wheel rolls along. (c) Find the x-values for which the particle is in contact with the surface upon which the wheel rolls. (d) What is the acceleration of the particle? (e) At what times is the particle not moving, and where is the particle on the rim related to the horizontal surface at these times?

(a) What are the velocity and speed of the particle?

The velocity of the particle as a function of time is given by

$$\vec{v} = \frac{d\vec{r}}{dt} = (10 - 10\sin 2t)i - (10\cos 2t)j$$

And as the speed is the magnitude of the velocity, given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10 - 10\sin 2t)^2 + (10\cos 2t)^2}$$

Or $$v = \sqrt{v_x^2 + v_y^2} = 10\sqrt{1 - 2\sin 2t + 1} = 10\sqrt{2}\sqrt{1 - \sin 2t} \quad \text{-------------- (1)}$$

(b) Sketch the path that the particle traces out as the wheel rolls along.

The position vector of the point is given as

$$\vec{r} = (10t + 5\cos 2t)i + (5 - 5\sin 2t)j,$$

This is the component form of the vector and it means that x and y coordinates of the position of point is given by

$x = 10*t + 5 \cos 2t$

and $y = 5 - 5 \sin 2t$

The coordinates are varying with time and the minimum values of the y coordinate between 0 (for t = $3\pi/2$, $7\pi/2$, $11\pi/2$, $15\pi/2$ ….) and the maximum value is 10 m (for t = $\pi/2$, $5\pi/2$, $9\pi/2$, $13\pi/2$ …..).

This shows that the wheel is rolling over x axis.

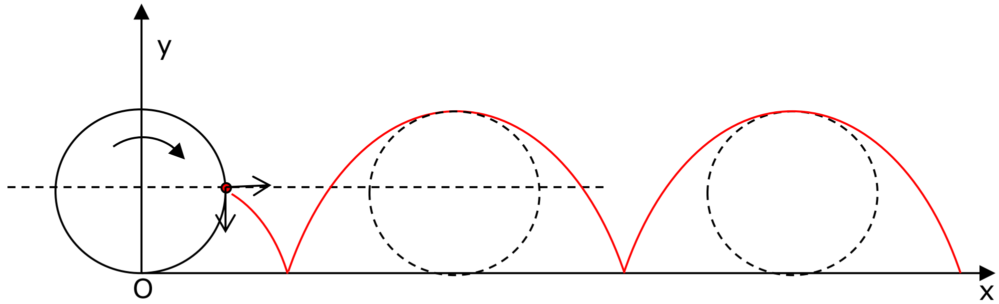
The initial position (at t = 0) of the particle is given by (x, y) = (5, 5) and the radius of the wheel will be 5m.

The path of particle is a curve given by the parametric equations

$x = 10*t + 5 \cos 2t$

And $y = 5 - 5 \sin 2t$

With time as the parameter and the curve is known as a cycloid. The sketch of the path is shown by red curve in figure 2.



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(c) Find the x-values for which the particle is in contact with the surface upon which the wheel rolls.

As from the initial position and the sketch it is clear that the particle will touch the surface (x axis) first after one fourth rotation and then after each complete rotation. In one complete rotation, the wheel covers a distance equal to its perimeter i.e. $2\pi R = 10\pi$ m = 31.4 m hence the particle will in contact with the surface at x positions

$2.5\pi,\ 12.5\ \pi,\ 22.5\ \pi,\ 32.5\ \pi,\ 42.5\ \pi,$

Or $x = (2.5 + 10n)\ \pi$ where n = 0, 1, 2, 3, 4 …..

(d) What is the acceleration of the particle?

The acceleration is the rate of change of the velocity and hence given by differentiating velocity w.r.t. time as

$$\vec{v} = \frac{d\vec{r}}{dt} = (10 - 10\sin 2t)i - (10\cos 2t)j$$

Gives $$\vec{a} = \frac{d\vec{v}}{dt} = (-20\cos 2t)i - (-20\sin 2t)j$$

Hence the magnitude of the acceleration is given by

$$a = \sqrt{(-20\cos 2t)^2 - (-20\sin 2t)^2} = 20\sqrt{(\cos 2t)^2 + (\sin 2t)^2} = 20\,\text{m/s}^2$$

As rolling is the combination of the uniform translation of the wheel along the surface and the rotation of the wheel about an axis passing through its center, the acceleration of the particle on the wheel is only corresponding to its rotation and hence it is the centripetal acceleration, constant in magnitude but varying in direction.

(e) At what times is the particle not moving, and how is the particle on the rim related to the horizontal surface at these times?

The particle will be at rest when it is in contact with the surface or at all times the y coordinates of the particle are zero and the particle is at the lowest point of the rim.

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