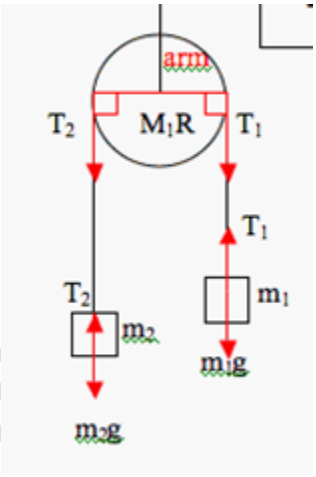
Q- Two blocks of mass m_1 = 4 kg and m_2 = 3 kg are attached to the ends of a string and passes over a pulley of mass M = 2 kg and radius 20 cm. Find angular acceleration of the pulley considering it as a uniform disk.

The system can be drawn as in the figure.



Let the magnitude of acceleration of m_1 and m_2 is a and the angular acceleration of the pulley is α.

The equations of motion are given by

$$m_1 a = T_1 - m_1 g \quad \text{---------- (1)}$$

$$m_2 a = m_2 g - T_2 \quad \text{---------- (2)}$$

And

$$T_2 R - T_1 R = I\alpha$$

Or $$T_2 - T_1 = \frac{I\alpha}{R} \quad \text{---------- (3)}$$

Here I is the moment of inertia of the pulley.

As the pulley is having mass M and the radius R, considering it as a uniform disk its moment of inertia is given by

$I = MR^2/2$

And as the angular acceleration is related to the linear acceleration by the relation

$\alpha = a/R$

Substituting these in equation (3) it becomes

Or $$T_2 - T_1 = \frac{1}{2} MR^2 \frac{a}{R^2}$$

Or $$T_2 - T_1 = \frac{1}{2} Ma \quad \text{---------- (3)}$$

Adding the three equations we have

$$m_2 g - m_1 g = \left(m_1 + m_2 + \frac{M}{2} \right) a$$

Or $$a = \frac{\left(m_2 - m_1 \right) g}{\left(m_1 + m_2 + \frac{M}{2} \right)}$$

Substituting the values, the magnitude of the acceleration a of m_1 and m_2 is given by

$$a = \frac{(3-4)*9.8}{(4+3+1)} = -1.225 m/s^2$$

And $\alpha = \frac{a}{R} = \frac{-1.225}{0.2m}$ = - 6.125 rad/s^2

Negative sign shows that the system will move in clockwise direction.