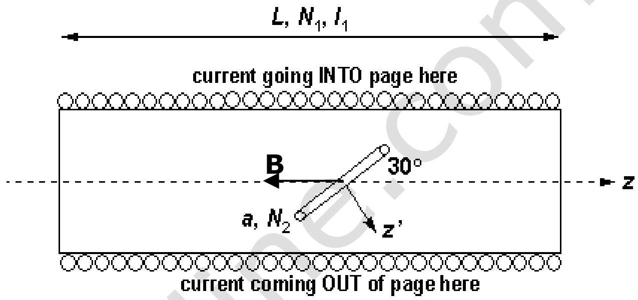
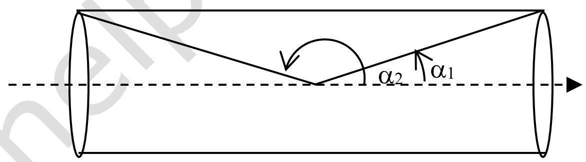
Q- A solenoid has length L = 30 cm, radius 6 cm, and N_1 = 5500 turns; its axis coincides with the z-axis. A circular conducting loop containing N_2 = 15 turns of radius a = 3 cm is **centered** inside the solenoid; the plane of the loop makes a 30° angle with respect to the z-axis. The loop has a net resistance of R = 0.015 Ω. The current in the windings of the solenoid is varying with time according to the expression $I_1(t)$ = 0.35 A + 0.75 (A/s) t. (When viewed from the positive z-direction, current traveling in the clockwise direction is considered positive.)

Calculate the induced current in the loop at t = 4 s.



The number of turns in the solenoid
$N_1 = 5500$
The length of the solenoid L =30 cm = 0.3 m
Hence the number of turns per unit length will be
$n = N_1/L$



The magnetic field B in a solenoid carrying current I_1 at point where the edges of the solenoid subtends angles α_1 and α_2 is given by

$$B = \frac{1}{2}\mu_0 n I_1(\cos\alpha_1 - \cos\alpha_2)$$

Or $$B = \frac{\mu_0 N_1 I_1}{2L}(\cos\alpha_1 - \cos\alpha_2)$$

As the loop is **centered** at the solenoid, the angle $\alpha_2 = 180^0 - \alpha_1$ and hence the field at the center of the solenoid is given by

$$B = \frac{\mu_0 N_1 I_1}{2L}\left(\cos\alpha_1 - \cos(180^0 - \alpha_1)\right) = \frac{\mu_0 N_1 I_1}{2L}(\cos\alpha_1 + \cos\alpha_1) = \frac{\mu_0 N_1 I_1 \cos\alpha_1}{L}$$

As the current in the coil is in clockwise direction as seen from + z direction, using the right hand rule the direction of magnetic field in the solenoid is towards the - z direction or to the left.

And hence the flux through the loop is given by

$$\phi_B = N_2(\vec{B}\bullet\vec{A}) = N_2\left(\frac{\mu_0 N_1 I_1 \cos\alpha_1}{L}\right)A\cos\theta = \frac{\mu_0 N_1 N_2 A I_1 \cos\alpha_1 \cos\theta}{L}$$

Where θ is the angle between the magnetic field vector and the area vector A

As the area vector is normal to the plane of the loop the angle between the field B and the area A vectors will be $\theta = 90^0 + 30^0 = 120^0$

Hence the EMF induced in the loop is given by faraday's law as

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{\mu_0 N_1 N_2 A\cos\alpha_1 \cos\theta}{L} * \frac{dI_1}{dt}$$

Now as $I_1(t)$ = 0.35 A + 0.75 (A/s) t

$$\frac{dI_1}{dt} = 0 + 0.75 = \frac{3}{4} \text{ A/s}$$

(Rate of change of current is independent of time)

Substituting in above equation we get

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{3\mu_0 N_1 N_2 A\cos\alpha_1 \cos\theta}{4L}$$

And the current in the loop as a function of time will be given by

$$I_2 = \frac{\varepsilon}{R} = -\frac{3\mu_0 N_1 N_2 A\cos\alpha_1 \cos\theta}{4LR}$$

Now here

$\mu_0 = 4\pi*10^{-7}$
$N_1 = 5500$
$N_2 = 15$
$A = \pi r^2 = 3.14*0.03^2 = 2.83*10^{-3}$ m^2
$\text{Cos } \alpha_1 = \dfrac{15}{\sqrt{15^2 + 6^2}} = 0.9285$
$\text{Cos } \theta = \cos 120^0 = -0.5$
$t = 4$ s
$L = 0.3$ m
And $R = 0.015\ \Omega$

Substituting all data, we get the current at t = 4 s as

$$I_2 = -\frac{3*4\pi*10^{-7}*5500*15*2.83*10^{-3}*0.9285*(-0.5)}{4*0.3*0.015}$$

Or $I_2 = 0.0227$ A

Hence current in the loop at any time is

I = 0.0227 A