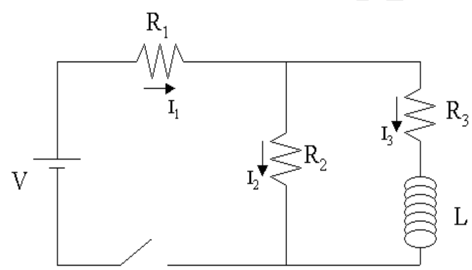
Q- Three resistors (R_1 = 120 Ohms, R_2 = 330 Ohms, and R_3 = 240 Ohms) and an ideal inductor (L = 1.6 mH) are connected to a battery (V = 9 V) through a switch as shown in the figure. The switch has been open for a long time before it is closed at t = 0. At what time t, does the current through the inductor (I_3) reach a value that is 63% of its maximum value?



Here as soon as the switch is closed the current starts flowing in the R_1 R_2 loop due to which there will be a potential difference across R_2 which force a current in the series of R_3 and the inductor L. Due to electromagnetic induction an EMF will be induced in the inductor which changes with time and hence the currents in both loop changes. Let at time t after the switch is closed, the currents in the three parts are I_1, I_2 and I_3 respectively.

According to junction law the charge will not accumulate at any junction we have

$$I_1 = I_2 + I_3 \qquad \text{---------------} \ (1)$$

As R_2 is in parallel with R_3 and L the potential difference across the two branches will be equal and this gives

$$I_2R_2 = I_3R_3 + L*(dI_3/dt) \qquad \text{---------------} \ (2)$$

As the EMF of the battery is dropped in the left loop, using loop rule we get

$$\varepsilon = I_1R_1 + I_2R_2 \qquad \text{---------------} \ (3)$$

Substituting the value of I_1 from equation 1 in 3 we get

$$\varepsilon = \left(I_2 + I_3\right)R_1 + I_2R_2 = I_2\left(R_1 + R_2\right) + I_3R_1$$

Or $$I_2 = \frac{\varepsilon - I_3R_1}{\left(R_1 + R_2\right)} \qquad \text{---------------} \ (4)$$

Substituting this value of I_2 in equation 2 we get

$$\frac{\varepsilon - I_3R_1}{\left(R_1 + R_2\right)} * R_2 = I_3R_3 + L\frac{dI_3}{dt}$$

Or $$L\frac{dI_3}{dt} = \frac{\varepsilon - I_3 R_1}{\left(R_1 + R_2\right)} * R_2 - I_3 R_3$$

Or $$L\frac{dI_3}{dt} = \frac{\varepsilon R_2 - I_3\left(R_1R_2 + R_1R_3 + R_2R_3\right)}{\left(R_1 + R_2\right)}$$

Or $$\frac{dI_3}{\varepsilon R_2 - I_3\left(R_1R_2 + R_1R_3 + R_2R_3\right)} = \frac{dt}{L\left(R_1 + R_2\right)}$$

Integrating with proper limits we have (initially current in the inductance is zero)

$$\int_0^{I_3}\frac{dI_3}{\varepsilon R_2 - I_3\left(R_1R_2 + R_1R_3 + R_2R_3\right)} = \int_0^t \frac{dt}{L\left(R_1 + R_2\right)}$$

Or $$-\frac{1}{\left(R_1R_2 + R_1R_3 + R_2R_3\right)}\ln\left[\frac{\varepsilon R_2 - I_3\left(R_1R_2 + R_1R_3 + R_2R_3\right)}{\varepsilon R_2}\right] = \frac{t}{L\left(R_1 + R_2\right)}$$

Or $$\ln\left[\frac{\varepsilon R_2 - I_3\left(R_1R_2 + R_1R_3 + R_2R_3\right)}{\varepsilon R_2}\right] = -\frac{\left(R_1R_2 + R_1R_3 + R_2R_3\right)t}{L\left(R_1 + R_2\right)}$$

Or $$\left[1 - \frac{I_3\left(R_1R_2 + R_1R_3 + R_2R_3\right)}{\varepsilon R_2}\right] = e^{-\frac{\left(R_1R_2 + R_1R_3 + R_2R_3\right)*t}{L\left(R_1 + R_2\right)}}$$

Or $$I_3 = \frac{\varepsilon R_2}{\left(R_1R_2 + R_1R_3 + R_2R_3\right)}\left[1 - e^{-\frac{\left(R_1R_2 + R_1R_3 + R_2R_3\right)*t}{L\left(R_1 + R_2\right)}}\right]$$

The elements in the circuit shown above have the following values: ε = 9 V, L = 1.6 mH, R_1 = 120 Ω, R_2 = 330 Ω and R_3 = 240 Ω.

$$I_3 = \frac{9*330}{120*330+120*240+330*240}\left[1 - e^{-(120*330+120*240+330*240)t}\right]$$

Or $$I_3 = 0.021\left[1 - e^{-147600*t}\right]$$

The current is maximum at long time and will be 0.021 A. hence the required time t is given by

$$0.021 * \frac{63}{100} = 0.021\left[1 - e^{-147600*t}\right]$$

Or $\frac{63}{100} = 1 - e^{-147600*t}$

Or $e^{-147600*t} = 1 - \frac{63}{100}$

Or $e^{-147600*t} = 0.37$

Or $e^{147600*t} = 2.703$

Or $147600 * t = \ln 2.703$

Or $147600 * t = 0.994$

Or $t = 6.74*10^{-6}$ s