Q- The earth orbits the sun at a distance of about $1.5*10^{11}$ m in about 365 days ($3.15*10^7$ s).
(a) What is the earth's speed along its orbital path?
(b) If the mass of the earth is $6*10^{24}$ kg, find the force exerted on it by the sun to keep it in orbit.
(c) What must be the mass of the sun to exert this much force on the earth?

The time taken T to make one complete rotation by a body moving in a circular orbit is called the time period and here it is

$T = 3.15*10^7$ sec.

The angle turned by a body about the center of its circular path per unit time is called its angular velocity and is denoted by ω. In one complete orbit the angle turned is 2π radians; hence the angular velocity of the earth in its orbit is given by

$$\omega = \frac{2\pi}{T}$$

The angular velocity and the linear speed of a particle in circular orbit are related by

$$v = \omega R = \frac{2\pi R}{T}$$

Solutions:
(a) In one orbit the total distance covered by a particle is equal to the perimeter of the circular path given by $2\pi R$, where R is the radius of circular path. Thus the linear speed of the earth (the distance covered per unit time) is given by

$$v = \frac{2\pi R}{T} = \frac{2*3.14*1.5*10^{11}}{3.15*10^7} = 2.99 * 10^4\ m/s$$

(b) To make a body of mass m move on a circular path of radius R with constant speed v, a force towards the center of circular path is required. As this force is always points to the center of the circular path called centripetal force and its magnitude is given by

$$F_{CP} = \frac{mv^2}{R}$$

To move the earth on the circular path in its orbit the centripetal force required will be given by

$$F_{CP} = \frac{m_e v^2}{R} = \frac{6*10^{24}(2.99*10^4)^2}{1.5*10^{11}} = 3.58 * 10^{22} N$$

Thus the force exerted on the earth by the sun is $3.58*10^{22}$ N.

(c) The centripetal force required to move the earth on circular orbit is exerted by the sun in form of gravitational force of attraction between the sun and the earth.
The gravitational force of attraction between two bodies of mass m_1 and m_2 at a distance R is given by Newton's law of universal gravitation as

$$F = \frac{Gm_1m_2}{R^2}$$

The gravitational force between the sun and the earth acts as centripetal force and hence we get

$$\frac{Gm_sm_e}{R^2} = F_{CP}$$

Substituting the values we get

$$\frac{6.67*10^{-11}*m_s*6*10^{24}}{(1.5*10^{11})^2} = 3.58 * 10^{22}$$

Or $$m_s = \frac{3.58*10^{22}*(1.5*10^{11})^2}{6.67*10^{-11}*6*10^{24}} = 2.01 * 10^{30} kg$$