

Q- Two parallel conducting plates are separated by the distance d , and the potential difference between the plates is maintained at the value V . A slab of dielectric with constant K and a uniform thickness $t < d$ is inserted between the plates and parallel to them. Find the electric field \vec{E} and displacement \vec{D} both in the dielectric and the air inbetween. Neglect edge effects.

Let the distance of the dielectric slab from plate A be x as the thickness of the slab is t , the distance of plate B from the other surface of the slab will be $(d-x-t)$.

Let the magnitude of field strength between the plate A and the slab is E .

As the potential difference between two points separated by a distance l in a uniform field of strength E is given by $V = -\vec{E} \cdot \vec{l}$, the potential difference V_1 between plate A and the slab will be given by

$$V_1 = E * x \quad \text{----- (1)}$$

Now due to the negative polarization charges the field strength in the slab will be reduced and its magnitude is given by

$$E_s = E/k$$

As this field will continue till thickness t the potential difference between the two surfaces of the slab is given by

$$V_2 = E_s * t = (E/k) * t \quad \text{----- (2)}$$

As because of the positive polarization charges on the second surface on the slab the field between the second surface of the slab and plate B becomes E again, the potential difference between them will be given by

$$V_3 = E * (d-x-t) \quad \text{----- (3)}$$

The total potential difference between the two plates is thus given by

$$V = V_1 + V_2 + V_3$$

Substituting the values from equations (1), (2) and (3) we get

$$V = E * x + \frac{E}{k} * t + E(d - x - t)$$

or
$$V = \frac{E}{k} * t + E(d - t)$$

gives
$$E = \frac{V}{(d-t) + \frac{t}{k}}$$

Or
$$E = \frac{V * k}{(d-t) * k + t}$$

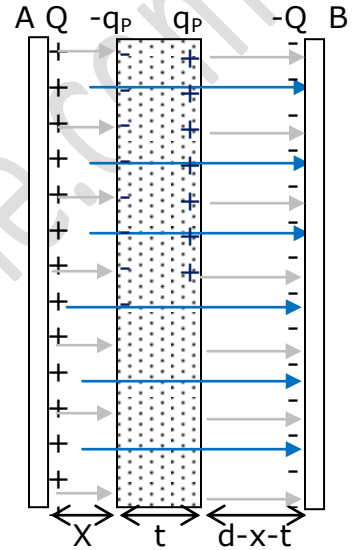
The direction of this field will be from higher potential plate to the lower potential plate hence if the potential is decreasing along the positive x direction we can write

$$\vec{E} = \frac{V * k}{(d-t) * k + t} (\hat{i}) \quad \text{----- (4)}$$

The displacement vector D is given by the relation $\mathbf{D} = k * \mathbf{E}$ hence

(A) In the air the dielectric constant is 1 and thus both E vector and D vector are same and given by

$$\vec{E}_v = \vec{D}_v = \frac{V * k}{(d-t) * k + t} (\hat{i})$$



(B) In the dielectric the electric field is reduced to E_v/k due to the polarization charges and hence given by

$$\vec{E} = \frac{\vec{E}_v}{k} = \frac{V}{(d-t).k+t} (\hat{i})$$

And the displacement vector is given by

$$\vec{D} = k\vec{E} = k \frac{\vec{E}_v}{k} = \vec{E}_v = \frac{V.k}{(d-t).k+t} (\hat{i})$$

Hence from vacuum to the dielectric the electric field vector is not continuous but the displacement vector is continuous.
