Q- Q- A particle executes linear harmonic motion about the point x = 0. At t =0, it has a displacement x = 0.37 cm and zero velocity. The frequency of oscillation is 0.25 Hz. Determine
(a) The period (b) the angular frequency (c) the amplitude (d) the displacement at time t (e) fthe velocity at time t (f) the maximum speed (g) the maximum acceleration (h) the displacement at t = 3.0 s, and (i) the speed at t = 3.0 s.

The equation for the displacement x of a particle executing simple harmonic motion in a straight line to and fro about its mean position, as a function of time t is given by

$$x = A\sin(\omega t + \phi_0) \qquad \text{------------------ (1)}$$

Where A is the amplitude (the maximum displacement of the particle during motion), ω is the angular frequency (the angular velocity of reference particle) and ϕ_0 is the phase constant or initial phase angle of the particle at t = 0

Differentiating equation 1 with respect to t we will get the velocity v of the particle as a function of time t as

$$v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi_0)$$

or $$v = A\omega\cos(\omega t + \phi_0) \qquad \text{------------------- (2)}$$

Differentiating again we get acceleration of the particle as a function of time

$$a = \frac{dv}{dt} = -A\omega^2\sin(\omega t + \phi_0)$$

or $$a = -A\omega^2\sin(\omega t + \phi_0) \qquad \text{------------------- (3)}$$

Now from our question at t = 0 the displacement is x = 0.37 cm and velocity v is zero substituting these data in equations 1 and 2 we get

$$0.37 = A\sin(\omega * 0 + \phi_0)$$

Gives $A \sin\phi_0 = 0.37$ ------------------ (4)

And $$0 = A\omega\cos(\omega * 0 + \phi_0)$$

Gives $\cos \phi_0 = 0$
or $\phi_0 = \pi/2$ ------------------- (5)

Substituting in equation 4 we get
$A \sin \pi/2 = 0.37$
Or $A = 0.37$ cm ------------------- (6)

Hence the equation of motion of the particle is given by
$x = (0.37\text{cm}) \sin (\omega t + \pi/2)$ ------------------ (7)

As the frequency n of oscillation is 0.25 Hz the angular frequency is given by

$$\omega = 2\pi * n = 2\pi * 0.25 = \frac{\pi}{2} \text{ rad/s}$$

And hence the equations for displacement, velocity and acceleration of the particle as a function of time can be rewritten as

$$x = (0.37cm)\sin\left(\frac{\pi}{2}t + \frac{\pi}{2}\right)$$

or $$x = (0.37cm)\sin\frac{\pi}{2}(t+1)$$ --------------------- (A)

$$v = (0.37cm)\left(\frac{\pi}{2}s^{-1}\right)\cos\frac{\pi}{2}(t+1)$$ --------------------- (B)

and $$a = -(0.37cm)\left(\frac{\pi}{2}s^{-1}\right)^2 \sin\frac{\pi}{2}(t+1)$$ ---------------------- (c)

Now to the question

(a) The time period T is the time taken by the particle in one complete oscillation and the frequency n is the number of oscillation per unit time means in one second hence both are related as

$$T = \frac{1}{n}$$

As the frequency of the particle is 0.25 Hz hence the time period will be

$$T = \frac{1}{n} = \frac{1}{0.25} = 4s$$

(b) The angular frequency is the angular velocity with which reference particle (imaginary) make one complete revolution in one time period and hence given by

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2} = 1.57.rad/s$$

(c) The amplitude is the maximum displacement of the particle from its mean position during its motion and as the displacement is given by equation (A)

$$x = (0.37cm)\sin\frac{\pi}{2}(t+1)$$ --------------------- (A)

x will be max when $\sin\frac{\pi}{2}(t+1)$ will have its maximum value that is 1 and hence the amplitude will be

A = 0.37 cm = $3.7*10^{-3}$ m.

(d) The displacement at any time t is given by equation (A) hence

$$x = (0.37cm)\sin\frac{\pi}{2}(t+1)$$

(e) The velocity of the particle as a function of time is given by equation (B) as

$$v = (0.37cm)\left(\frac{\pi}{2}s^{-1}\right)\cos\frac{\pi}{2}(t+1)$$ --------------------- (B)

(f) The speed of the particle will be maximum when the value of cosine term in the expression of velocity is maximum i.e. 1 hence

$$v_{\max} = (0.37cm)\left(\frac{\pi}{2}s^{-1}\right) = 0.37*1.57 = 0.581cm/s$$

(g) The acceleration of the particle is given by the equation (C) as

$$a = -(0.37cm)\left(\frac{\pi}{2}s^{-1}\right)^2 \sin\frac{\pi}{2}(t+1) \qquad \text{---------------------- (c)}$$

Hence the maximum acceleration is given by

$$a_{\max} = -(0.37cm)\left(\frac{\pi}{2}s^{-1}\right)^2 = -0.913 \text{ cm/s}^2$$

The magnitude of this acceleration is 0.913 cm/s^2. The negative sign correspond to the direction. Acceleration always points to the mean position.

(h) The displacement at any time is given by equation (A)

$$x = (0.37cm)\sin\frac{\pi}{2}(t+1)$$

Hence at time t = 3.0 s the displacement of the particle is given by

$$x(3.0) = (0.37cm)\sin\frac{\pi}{2}(3.0+1.0) = (0.37cm)\sin(2\pi) = 0$$

Thus the displacement of the particle at t =3 second is zero or the particle is at mean position.

(I) The speed of the particle is given by equation (B)

$$v = (0.37cm)\left(\frac{\pi}{2}s^{-1}\right)\cos\frac{\pi}{2}(t+1)$$

Hence at t = 3.0 s

$$v(3.0) = (0.37cm)\left(\frac{\pi}{2}s^{-1}\right)\cos\frac{\pi}{2}(3+1) = (0.37cm)\left(\frac{\pi}{2}s^{-1}\right)\cos(2\pi)$$

Or $v(3.0) = (0.37cm)(1.57s^{-1})*1 = 0.581$ cm/s (the maximum)