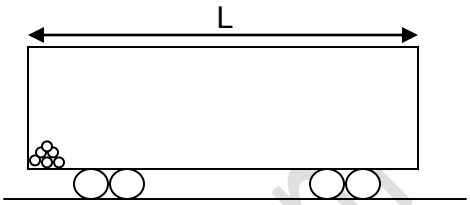
Q- A closed railroad car of length L and mass M on a frictionless track contains a cannon firing cannonballs of total mass m from one end to the other. Assume that all the cannonballs each move a total distance L. Find a formula for the resulting displacement d of the railroad car. Does your formula have the right behavior for M>>m and for m>>M?



The problem is based on the principle of center of mass. As before and after the cannonballs are fired, they are at rest and as there is no external horizontal force on the system of the car and the balls, the center of mass of the whole system remains at the same point i.e. the net displacement of the center of mass of the system will be zero.

As the balls are fired from one end to the other, the distance moved by all balls relative to the car should be L. If the displacement of the center of mass of the car is Δx_C to the left then the displacement of the balls relative to the earth will be $\Delta x_b = L - \Delta x_C$ to the right.

Now according to the concept of center of mass the displacement of the center of mass of the whole system is given by

$$\Delta x_{CM} = (m_1 * \Delta x_1 + m_2 * \Delta x_2)/(m_1 + m_2)$$

Or $$0 = [M(-\Delta x_C) + m\,\Delta x_b]/(M + m)$$

Gives $$M(-\Delta x_C) + m\,\Delta x_b = 0$$

Or $$\Delta x_C = m\,\Delta x_b/M$$

Substituting the values we have

$$\Delta x_C = m*(L - \Delta x_C)/M$$

Gives $$\mathbf{\Delta x_C = m*L/ (M + m)}$$

Now if M >> m, m/(M + m) is very small and the displacement of the car is negligible. And if m >> M, m/(M + m) is very near to 1 and the displacement of the car is L means the displacement of the balls is very small.
The formula is giving correct results in both conditions.