

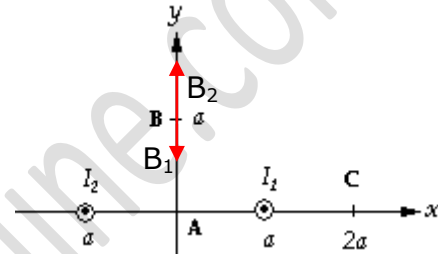
Q- Two infinite wires run parallel to the z-axis, which points out of the page. One wire carries a current $I_1 = 2$ A and intersects the x-axis at $x = a = 3$ cm. The second wire carries current $I_2 = 5$ A and intersects the x-axis at $x = -a$. Both currents flow in the +z direction.

- (a) Calculate the net magnetic field due to I_1 and I_2 at Point A (the origin).
- (b) Calculate the net magnetic field due to I_1 and I_2 at Point B a distance a above the origin on the y-axis.
- (c) Calculate the net magnetic field due to I_1 and I_2 at Point C a distance $2a$ to the right of the origin on the x-axis.

(a) The magnetic field due to long straight wire carrying a current I at a distance r from it is given by

$$B = \frac{\mu_0 I}{2\pi r}$$

The direction of the magnetic field at a point is given by the right hand rule.



Now here the field at A due to the wire carrying current I_1 will be

$$B_1 = \frac{\mu_0 I_1}{2\pi a} = \frac{2 * 10^{-7} * 2}{0.03} = 1.33 * 10^{-5} \text{ T}$$

According to right hand rule, the direction of this current is outward and at A the field will be downward means negative y direction and hence

$$\vec{B}_1 = -1.33 * 10^{-5} \hat{j} \text{ T} \quad \text{----- (1)}$$

The field at A due to the wire carrying current I_2 will be

$$B_2 = \frac{\mu_0 I_2}{2\pi a} = \frac{2 * 10^{-7} * 5}{0.03} = 3.33 * 10^{-5} \text{ T}$$

According to right hand rule, the direction of this current is outward and at A the field will be upward means positive y direction and hence

$$\vec{B}_2 = 3.33 * 10^{-5} \hat{j} \text{ T} \quad \text{----- (2)}$$

Hence the total field at A is given by

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = (-1.33 * 10^{-5} + 3.33 * 10^{-5}) \hat{j} \text{ T}$$

Hence the field at A is $2.0 * 10^{-5}$ T and in positive y direction

(b) Now here the distance of each wire is $r = \sqrt{a^2 + a^2} = a\sqrt{2}$ and hence field at B due to the wire carrying current I_1 will be

$$B_1 = \frac{\mu_0 I_1}{2\pi a\sqrt{2}} = \frac{2 * 10^{-7} * 2}{0.03\sqrt{2}} = 9.43 * 10^{-6} \text{ T} \quad \text{T}$$

According to right hand rule, the direction of this current is outward and at A the field will be making 45° with negative x direction and hence in component form can be written as

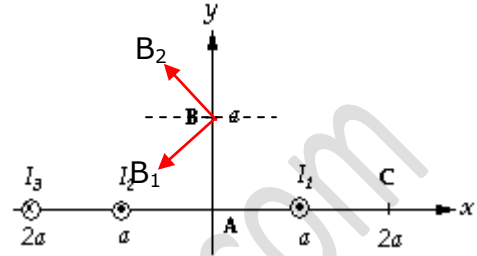
$$\vec{B}_1 = 9.43 * 10^{-6}(-\cos 45^\circ \hat{i} - \cos 45^\circ \hat{j}) = -6.67 * 10^{-6} (\hat{i} + \hat{j}) \text{ T} \quad \text{----- (3)}$$

The field at B due to the wire carrying current I_2 will be

$$B_2 = \frac{\mu_0 I_2}{2\pi a \sqrt{2}} = \frac{2 * 10^{-7} * 5}{0.03 \sqrt{2}} = 2.36 * 10^{-5} \text{ T}$$

According to right hand rule, the direction of this current is also outward and at B the field will be making 45° with positive y direction and hence

$$\vec{B}_2 = 2.36 * 10^{-5}(-\sin 45^\circ \hat{i} + \cos 45^\circ \hat{j}) = 1.67 * 10^{-5}(-\hat{i} + \hat{j}) \text{ T} \quad \text{----- (4)}$$



Hence using equation 3 and 4 total field at B is given by

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = -6.67 * 10^{-6} (\hat{i} + \hat{j}) + 1.67 * 10^{-5}(-\hat{i} + \hat{j})$$

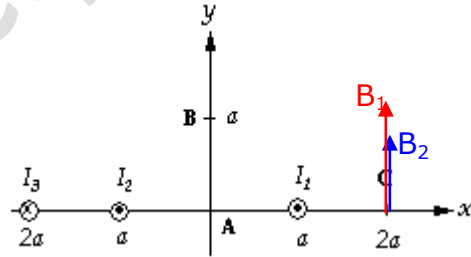
Or $\vec{B} = -2.34 * 10^{-5} \hat{i} + 1.003 \hat{j}$

(c) The distance between the wire carrying current I_1 and point C is $r_1 = 2a - a = a$ and hence the magnetic field due to this wire at C is given by

$$B_1 = \frac{\mu_0 I_1}{2\pi a} = \frac{2 * 10^{-7} * 2}{0.03} = 1.33 * 10^{-6} \text{ T}$$

According to right hand rule the current is outward and point C is to the right of the wire the field will be upward means to the positive y direction. Hence the field due to this wire is given by

$$\vec{B}_1 = 1.33 * 10^{-5} \hat{j} \text{ T} \quad \text{----- (5)}$$



The distance between the wire carrying current I_2 and point C is $r_2 = 2a + a = 3a$ and hence the magnetic field due to this wire at C is given by

$$B_2 = \frac{\mu_0 I_1}{2\pi a} = \frac{2 * 10^{-7} * 5}{3 * 0.03} = 1.11 * 10^{-5} \text{ T}$$

According to right hand rule the current is outward and point C is to the right of the wire the field will be upward means to the positive y direction. Hence the field due to this wire is given by

$$\vec{B}_1 = 1.11 * 10^{-5} \hat{j} \quad \text{----- (6)}$$

Hence the resultant field at point C due the two wires using equations 5 and 6 is

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = 1.33 * 10^{-5} \hat{j} + 10^{-5} \hat{j} = 2.44 * 10^{-5} \hat{j} \text{ T}$$