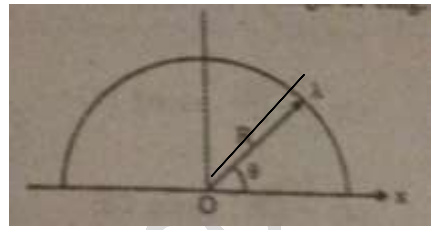
Q- (a) Linear charge density of a half ring varies with q as $\lambda = \lambda_0 \cos\theta$. Find total charge on the ring.

(b) If the linear charge density of a wire of length L depends on the distance x from its one end as $\lambda = \frac{\lambda_0 x}{L}$, find total charge of the wire.



(a) Consider a very small arc between angle θ and $\theta + d\theta$ subtending angle $d\theta$ at the center of the semicircle. The length of this element will be R*dq and thus the charge on this element will be

$$dq = \lambda R\, d\theta$$

Or $$dq = \lambda_0 \cos\theta\ R\, d\theta$$

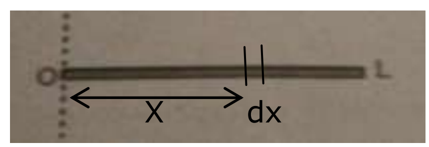
Thus the total charge on the half ring can be calculated by integrating the charge element for $\theta = 0$ to $\theta = \pi$ as

$$q = \int dq = \lambda_0 R \int_0^{\pi} \cos\theta\ d\theta$$

Or $$q = \lambda_0 R\, [\sin\theta]_0^{\pi}$$

Or $$q = \lambda_0 R\, [\sin\pi - \sin 0] = 0$$

(b)



Consider a small length element of length dx at a distance x from the end O.

The charge on this element will be

$$dq = \lambda\ dx$$

Or $$dq = \frac{\lambda_0 x}{L}\, dx$$

Thus the total charge on the wire can be calculated by integrating the charge element for x = 0 to x = L as

$$q = \int dq = \frac{\lambda_0}{L} \int_0^{L} x\, dx$$

Or $$q = \frac{\lambda_0}{L}\left[\frac{x^2}{2}\right]_0^{L}$$

Or $$q = \frac{\lambda_0}{L}\left[\frac{L^2}{2} - 0\right] = \frac{\lambda_0 L}{2}$$

(In part b the charge is varying linear and hence average charge density will be $\lambda_0/2$ and thus the total charge will be $\lambda_0 L/2$)