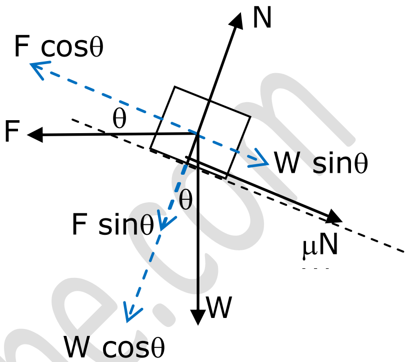
Q- A body of mass 80 kg is to be pushed upward along an inclined plane making angle 30^0 with horizontal. The coefficient of friction between the body and the plane is 0.2. Find the smallest force required if applied
(a) Horizontally
(b) Along the incline

The forces acting on the body are
(1) Its weight W = 80*g vertically down
(2) The normal force of the surface N
(3) The friction force which is along the inclined surface. As with the minimum force the body will just at the verge of moving up the incline, the friction will be down the incline and its magnitude will be μN.
(4) The force applied F.



The free body diagram for the body is shown.

Resolving the weight W and the force applied F, along the incline and normal to incline the components are shown in the free body diagram and in place of these two forces we will now consider their components.

As the body is in equilibrium, the forces along and normal to the incline are balanced separately.

Considering forces along the incline we get the relation

$$Wsin\theta + \mu N - Fcos\theta = 0 \quad \text{---------------------} \ (1)$$

And for the forces normal to the incline we get

$$N - Wcos\theta - Fsin\theta = 0$$

Or $N = Fsin\theta + Wcos\theta$

Substituting this value of N in equation (1) we get

$$Wsin\theta + \mu(Fsin\theta + Wcos\theta) - Fcos\theta = 0$$

Or $Wsin\theta + \mu * Fsin\theta + \mu * Wcos\theta - Fcos\theta = 0$

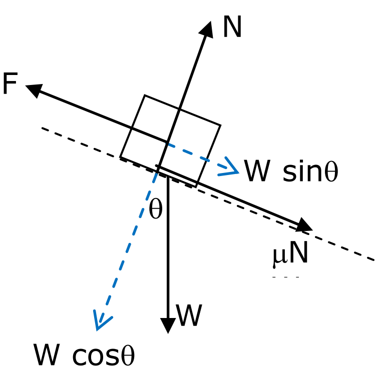
Or $F(cos\theta - \mu sin\theta) = W(sin\theta + \mu cos\theta)$

Or $F = \frac{W(sin\theta + \mu cos\theta)}{(cos\theta - \mu sin\theta)}$

Substituting the values given in the question

$$F = \frac{80*9.8*(sin30^0 + 0.2\ cos30^0)}{(cos30^0 - 0.2\ sin30^0)} = \frac{80*9.8*(0.500+0.173)}{(0.866-0.1)} = 689\ N$$

(b) The forces acting on the body are
(1) Its weight W = 80*9.8 N, vertically down
(2) The normal force of the surface N
(3) The friction force which is along the inclined surface. As with the minimum force F the body will just at the verge of moving up the incline, the friction will be down the incline and its magnitude will be □N.



(4) The force applied F along the incline.

The free body diagram for the body is shown.

Resolving the weight W along the incline and normal to incline the components are shown in the free body diagram.

As the body is in equilibrium, the forces along and normal to the incline are balanced separately.

Considering forces along the incline we get the relation

$$W sin\theta + \mu N - F = 0 \qquad \text{--------------------- (1)}$$

And for the forces normal to the incline we get

$$N - W cos\theta = 0$$

Or $N = W cos\theta$

Substituting this value of N in equation (1) we get

$$W sin\theta + \mu * W cos\theta - F = 0$$

Or $F = W(sin\theta + \mu cos\theta)$

Substituting the values given in the question

$$F = 80 * 9.8 * (sin30^0 + 0.2\, cos30^0) = 80 * 9.8 * (0.500 + 0.173) = 528\, N$$