Q- A hypodermic syringe is attached to a needle that has an internal radius of 0.3 cm and a length of 3 cm. The needle is filled with a solution of viscosity $2*10^{-3}$ Pa.s to be injected into a vein at a gauge pressure of 16.5 mm of Hg.

(a) What must be the pressure of the fluid in the syringe be in order to inject the solution at a reate of 0.25 mL/s?

(b) What force must be applied to the plunger which has an area of 1 cm^2?

The flow of fluid through a capillary is given by Poiseuille's equation which is given as

$$\frac{dQ}{dt} = \frac{\pi R^4 \Delta P}{8\eta L}$$

Where dQ/dt is rate of flow i.e. the volume flowing per unit time, R is the radius of the tube, Δp is the pressure difference at the two ends, L is the length of the tube and η is the coefficient of viscosity of the fluid.

(a) The Poiseuille's equation will give

$$\Delta P = \frac{8\eta L}{\pi R^4} * \frac{dQ}{dt}$$

Here R = 0.3 mm = $3*10^{-4}$ m L = 3.00 cm = $3.00*10^{-2}$ m
$\eta = 2*10^{-3}$ Pa.s dQ/dt = 0.25 mL/s = $2.5*10^{-7}$ m^3/s

Now if the pressure inside the syringe is P_1 and that in vain is P_2 then the pressure difference will be given by

$$P_1 - P_2 = \frac{8*2*10^{-3}*3*10^{-2}}{3.14*(3*10^{-4})^4} * 2.5*10^{-7} = 4.7*10^3\ Pa$$

Now the pressure in the vain (gauge pressure)

$P_2 = 16.5*10^{-3}*13.6*10^{3}*9.8$ Pa = $2.20*10^3$ Pa

Hence the gauge pressure of the fluid in the syringe is given by

$P_1 = P_2 + \Delta P = 2.20*10^3 + 4.7*10^3 =$ **$6.9*10^3$ Pa**

The absolute pressure in the syringe = gauge pressure + Atm pressure

= $6.9*10^3 + 1.01*10^5 = 1.08*10^5$ Pa

(b) As the flow rate in the syringe is very small the force required to create this excess pressure is given by

F = P*A = ($6.9*10^3$ Pa)*($1*10^{-4}$ m^2) = 0.**69 N**