

Q- A closed end organ pipe is sounded near a guitar causing one of the strings to vibrate with large amplitude. The string is 80% as long as the organ pipe. If both vibrate at their fundamental frequency, calculate the ratio of the speed of wave on the string to the speed of sound in air.

Solution:

This phenomenon is called resonance. When a body vibrates with frequency equal to natural frequency of another nearby body the second body starts vibrating with large amplitude. Hence the frequency of vibration of both the pipe and the wire will be equal. (say n)

When a standing wave is formed in a closed organ pipe in fundamental mode, the open end is antinode and the closed end is a node. As the distance between consecutive node and antinode is $\lambda/4$, the wavelength of the sound is four time the length of the tube i.e.

$$\lambda = 4l$$

As the speed of sound in the medium is given by the product of frequency and wavelength we get the speed of sound in air c_1 as

$$c_1 = n\lambda = n \cdot 4l \quad \text{----- (1)}$$

Where n is the frequency of sound and l is the length of the pipe.

The length of the wire is 80% of the length of the pipe, hence the length of the wire will be $(80/100)l = 4l/5$

Now when a wire stretched between two points vibrates in its fundamental mode, it's both ends are nodes and as the distance between two consecutive nodes is $\lambda'/2$ the wavelength of the standing wave in the string will be given by

$$\lambda' = 2(4l/5) = 8l/5$$

And the speed of the wave on the wire is given by

$$c_2 = n\lambda' = n(8l/5) \quad \text{----- (2)}$$

from the two equation we get

$$\frac{c_2}{c_1} = \frac{n(8l/5)}{n \cdot 4l} = \frac{2}{5}$$

Hence the required ratio is 2 : 5