Q- A 1.5 kg piston supported on a helical spring vibrates freely with a natural frequency of 3Hz. When oscillating within an oil filled cylinder the frequency of free oscillation is reduced to 2.90Hz. Assuming that the damping in the oil filled cylinder is viscous:

(a) Find the stiffness of the helical spring.

[Viscous damping means that the damping force is proportional to the velocity of the body or F_d = - c (dx/dt)].

The angular frequency of free natural oscillation of a body of mass m under force constant K is given by

$$\omega_n = \sqrt{\frac{K}{m}}$$

Here m is the mass of the body and K is the stiffness of the spring.

Now as the frequency and the angular frequency are related by f = $\omega/2\pi$, we get the natural frequency of free oscillation as

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$$

Substituting the values, we get

$$3 = \frac{1}{2*3.14}\sqrt{\frac{K}{1.5}}$$

Gives K = 532.42 N/m

(b) Find the damping ratio when in the oil filled cylinder.

The damping ratio ζ relates the natural frequency and the frequency with damping as

$$\omega_d = \omega_n * \sqrt{1-\zeta^2}$$

Or $$f_d = f_n * \sqrt{1-\zeta^2}$$

Substituting the values, we get

$$2.90 = 3.00 * \sqrt{1-\zeta^2}$$

Or $$\sqrt{1-\zeta^2} = \frac{2.90}{3.00}$$

Or $$1-\zeta^2 = 0.934$$

Gives $\zeta = 0.257$

As $\zeta < 1$, The oscillations are under damped

(c) Find the damping coefficient when in the oil filled cylinder.

The damping coefficient is related to the natural angular frequency as

$$c = 2m\omega_n\zeta = 2m(2\pi\, f_n)\,\zeta = \ 2 * 1.5(2\pi * 3) * 0.257 = 14.533 \text{ N.s/m}$$

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(d) In the oil filled cylinder, how many oscillations are required for the amplitude to be reduced to 1/20th of the initial displacement used to excite the system?

The amplitude as a function in the damped oscillation (viscous) is given by the relation

$$A(t) = A_0 e^{-(c/2m)\,t}$$

Where A_0 is the initial amplitude, b is damping constant and m is the mass of the object. Hence substituting the values we have

$$\frac{A_0}{20} = A_0 e^{-\left(\frac{14.533}{2*1.5}\right)t}$$

Or $e^{\left(\frac{14.533}{2*1.5}\right)t} = 20$

Gives $e^{(4.844)\,t} = 20$

Or 4.844*t = ln 20 = 2.996

Or t = 2.996/4.844 = 0.618 s

Hence the number of oscillations in this time will be

0.618*f_d = 0.618*2.90 = 1.792 oscillations.

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