Q- A well-insulated rigid cylinder is divided in to 2 compartments by a conducting piston that is free to move freely. Initially one side of the piston contains 1 m^3 of O_2 at 5 bar, 80^0C and the other side contains 1m^3 of N_2 at 7 bar, 20^0C. The piston is released and the system is allowed to come to equilibrium. Determine the final equilibrium temperature and pressure.

For the oxygen: Pressure P_1 =5 bar =$5*10^5$ Pa; Volume V_1 = 1 m^3; Temperature T_1 =80^0C =353 K

For Nitrogen: Pressure P_2 = 7 bar = $7*10^5$ Pa: Volume V_2 = 1 m^3; Temperature T_2 = 20^0C = 293 K

As the piston can move freely and conducting finally on either side the pressure P and the temperature T will be same.

Let the final volume of the oxygen be V then as the total volume of the cylinder is 2 m^3 the volume of nitrogen will be (2 – V) m^3.

As on either side the mass of the gases is not changing, using the relation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

For oxygen: $\frac{5*10^5*1}{353} = \frac{P*V}{T}$ ------------------------------ (1)

And for nitrogen: $\frac{7*10^5*1}{293} = \frac{P*(2-V)}{T}$ ------------------------------ (2)

Dividing the equations, we get

$$\frac{5*293}{7*353} = \frac{V}{2-V}$$

Gives 2471V = 2930 – 1465V

Or V = 2930/3936 = 0.744 m^3.

Hence the final volume of the oxygen is 8.33 m^3 and that of nitrogen will be 2 – V = 1.256 m^3.

Now the internal energy of n moles of a diatomic gas at temperature T is given by

$$U = nC_VT = \frac{5}{2}nRT = \frac{5}{2}PV$$

And as neither heat is transferred, nor work done by the gases outside the cylinder, total internal energy of the system remains same before and after the piston is released. Hence, we have

$$\frac{5}{2}P_1V_1 + \frac{5}{2}P_2V_2 = \frac{5}{2}PV + \frac{5}{2}P(2-V)$$

Gives $P_1V_1 + P_2V_2 = PV + P(2-V)$

Or $P_1V_1 + P_2V_2 = P*2$

Or $P = \frac{1}{2}(P_1V_1 + P_2V_2)$

Or P = 0.5($5*10^5*1 + 7*10^5*1$) = $6*10^5$ Pa = **6 bar**.

Substituting values of P and V in equation (1) we get the final temperature of the gases as

$$\frac{5*10^5*1}{353} = \frac{6*10^5*0.744}{T}$$

Or $T = \frac{6*10^5*0.744*353}{5*10^5} = 315.16\,K = \mathbf{42.16^0C}$

Hence the final temperature and pressure on either side are 42.16^0 C and 6 bar.