

Q- A solid wheel of mass 30 kg is rolled toward the left along a horizontal surface under the action of a horizontal force of magnitude 200 N, acting at the centre of the wheel. If the friction is sufficient to prevent slipping, what is the magnitude of linear acceleration of the centre of the wheel?

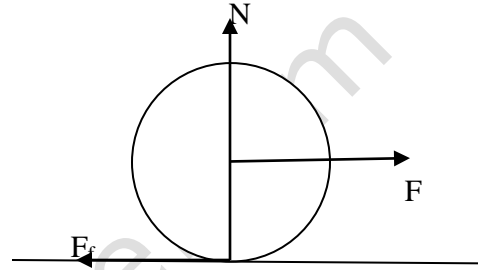
As there is no sliding, the friction force will be less than limiting friction force let it be  $F_f$ .

The acceleration  $a$  of centre of mass of the wheel will be given by using Newton's law as

$$F - F_f = ma \quad \text{----- (1)}$$

As the line of action of force  $F$  and the normal reaction  $N$  of the surface are passing through the center of mass of the wheel there moment about an axis passing through the center will be zero and hence the net moment (torque) is only due to the friction will be  $F_f \cdot R$ , hence writing the equation for rotational motion the angular acceleration of the wheel is given by

$$F_f \cdot R = I \cdot \alpha \quad \text{----- (2)}$$



Here  $I$  is the moment of Inertia of the wheel about axis passing through center of mass of the wheel equal to  $\frac{1}{2} mR^2$ .

Now as for pure rolling

$$a = \alpha R$$

Substituting the value of  $a$  and  $I$  in equation (2) we get

$$F_f \cdot R = \frac{1}{2} mR^2 \cdot (a / R)$$

$$\text{Or } F_f = \frac{1}{2} ma$$

Substituting in equation (1) we get

$$F - \frac{1}{2} ma = ma$$

$$\text{Gives } a = \frac{2F}{3m}$$

$$\text{Now } F = 200 \text{ N and } m = 30 \text{ kg}$$

$$\text{Thus } a = \frac{2 \cdot 200}{3 \cdot 30} = 4.44 \text{ m/s}^2$$

Alternatively:

The lowest point of the wheel is at rest, thus taking it as instantaneous axis of rotation, net torque about it is  $F \cdot R$  hence angular acceleration

$$\alpha = \frac{F \cdot R}{I} = \frac{F \cdot R}{\frac{3}{2} mR^2} = \frac{2F}{3mR}$$

$$\text{Hence, } a = \alpha R = \frac{2F}{3m}$$