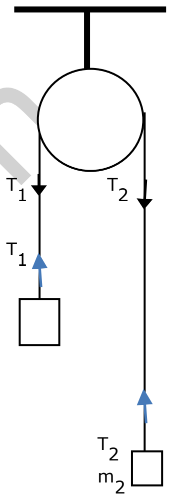
Q- A m_1 = 16.0 kg object and a m_2 = 12.5 kg object are suspended, joined by a cord that passes over a pulley with a radius of 10.0 cm and a mass of 3.00 kg. The cord has a negligible mass and does not slip on the pulley. The pulley rotates on its axis without friction. The objects start from rest 3.00 m apart. Treating the pulley as a uniform disk, determine the speeds of the two objects as they pass each other.

Answer:



The system can be considered as a single system rotating about the center of pulley. The external forces creating the torque are the weights of the masses only. The friction and the tension in the thread are the internal forces and torque due to the weight of the pulley (as it is passing through the point of suspension) is zero. Hence the torque on the system about the center of the pulley is given by

$\tau = m_1gR - m_2gR$ where R is the radius of the pulley.

Now consider the moment of inertia of the system. The masses m_1 and m_2 are moving on vertical straight lines and as we know that the length of the strings will not affect the angular acceleration (may be considered very small) the perpendicular distance of the masses from axis of rotation is to be taken for calculation of moment of inertia and hence the total moment of inertia is the moment of inertia due to the masses and due to the pulley. Given by

$I = m_1R^2 + m_2R^2 + (1/2)MR^2$

So the angular acceleration of the system is given by

$$\alpha = \tau / I = \frac{(m_1 - m_2)gR}{\left(m_1 + m_2 + M/2\right)R^2} = \frac{(m_1 - m_2)g}{\left(m_1 + m_2 + M/2\right)R} = \frac{3.5 \times 9.8}{30 \times 0.1} = 11.43 rad/s^2$$

Hence the linear acceleration of m_1(downward) and m_2(upward) are (in magnitude) given by

$a = \alpha R = 11.43 \times 0.1 = 1.14$ m/s^2.

Now as the two masses are approaching each other with same acceleration they will meet at mid-point and each travels a distance s = 3.0/2 = 1.5 m, and hence the velocity at this point is given by

$V^2 = u^2 + 2as = 0 + 2 \times 1.14 \times 1.5 = 3.42$.

Or $V = 1.85$ m/s

This is the required answer.

Note: For acceleration the equations can be written as follows

$m_1g - T_1 = m_1a$

$T_2 - m_2g = m_2a$ and

$T_1R - T_2R = (1/2)MR_2\ (a/R)$