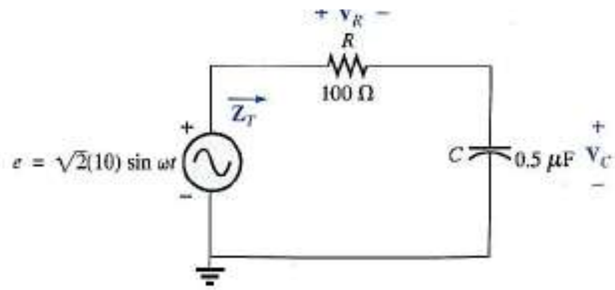
Q- A source of alternating voltage e = 10√2 sin ωt is connected to a resistor R = 100 Ω and a capacitor C = 0.5 μF in series.

(a) Plot impedance Z_T and phase difference between current and voltage θ_T versus frequency for a frequency range of zero to 10 kHz.

(b) Plot voltage across capacitor V_C versus frequency for the frequency range of part (a)

(c) Plot voltage across resistor V_R versus frequency for the frequency range of part (a).



The source voltage across the circuit is

V = 10 √2 sin ωt

Resistance R = 100 Ω

Capacitance C = 0.5 μF and

Angular frequency ω = 2π n

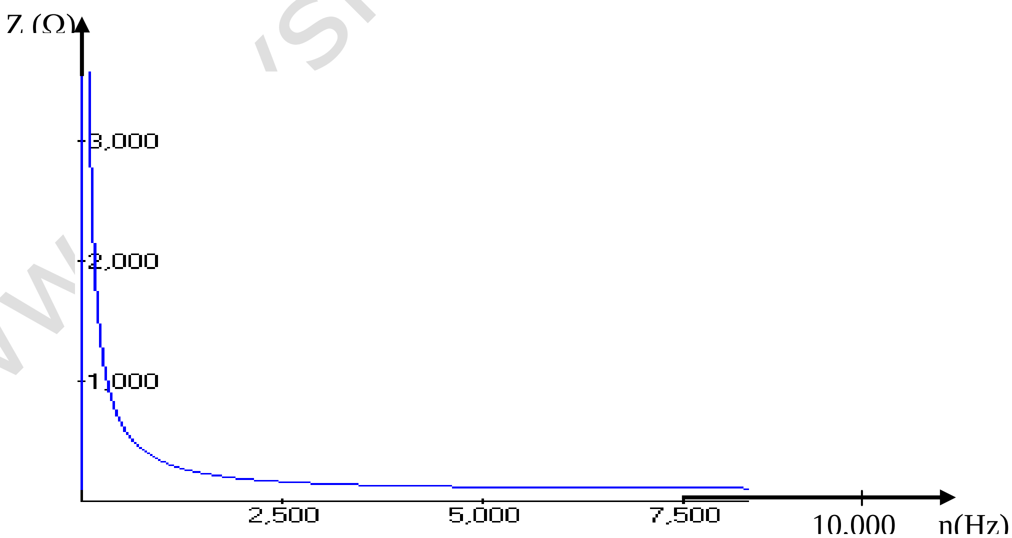
(a) Plot Z_T and θ_T versus frequency for a frequency range of zero to 10 kHz.

The impedance of the circuit is given by

$$Z = \sqrt{R^2 + {X_C}^2} = \sqrt{100^2 + \left(\frac{1}{C\omega}\right)^2} = \sqrt{100^2 + \left(\frac{1}{0.5*10^{-6}*2*3.14*n}\right)^2}$$

Or $$Z = \sqrt{100^2 + \left(\frac{1.01}{n^2}\right)} = \sqrt{10^4 + \left(\frac{1.01*10^{11}}{n^2}\right)}$$

For n =0, Z = ∞; for n = 10 k Hz, Z = 104.95 Ω. The Z-n plot will be given as bellow



The phase difference θ between the voltage and current will depend on the frequency and is given by

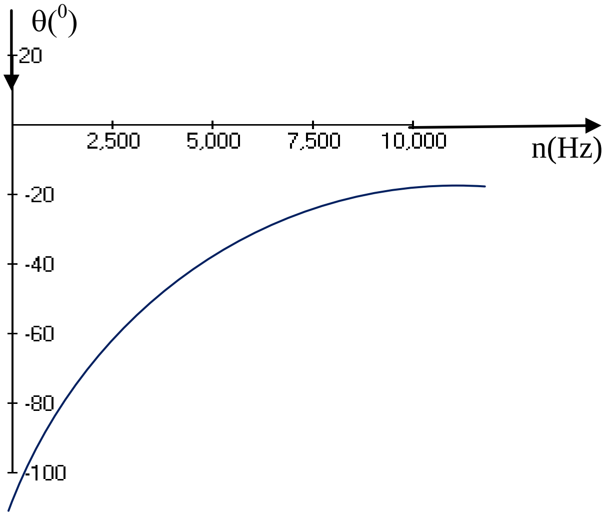
$$\theta = tan^{-1}\left(\frac{X_L - X_C}{R}\right) = tan^{-1}\left(\frac{-1}{R*C\omega}\right)$$

Or $$\theta = tan^{-1}\left(\frac{-1}{100*0.5*10^{-6}*2*3.14*n}\right)$$

Or $$\theta = tan^{-1}\left(-\frac{3184.7}{n}\right)$$

For n = 0, θ = - 90^0 and for n = 10000, θ = - 17.67^0

The qualitative plot in shown in the figure given below



(b) Plot V_C versus frequency for the frequency range of part (a).

The source voltage across the circuit is

$$V = 10\sqrt{2}\sin \omega t$$

Thus the effective voltage across the circuit is given by

$$V_{eff} = V_{rms} = \frac{10\sqrt{2}}{\sqrt{2}} = 10\,V$$

Hence the current in the circuit is given by

$$I_{eff} = \frac{V_{eff}}{Z} = \frac{10}{\sqrt{100^2+\left(\frac{1}{C\omega}\right)^2}}$$

And hence the voltage across the capacitor is given by

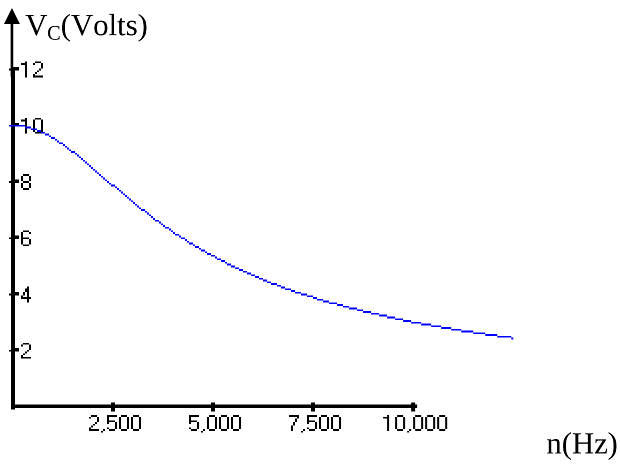
$$V_C = I_{eff} * X_C = \frac{10}{\sqrt{100^2+\left(\frac{1}{C\omega}\right)^2}} * \frac{1}{C\omega}$$

Or $$V_C = \frac{10}{\sqrt{10^4*(C\omega)^2+1}} = \frac{10}{\sqrt{10^4*(C*2\pi n)^2+1}} = \frac{10}{\sqrt{10^{-7}*n^2+1}}$$

Or $$V_C = \frac{10}{\sqrt{10^{-7}*n^2+1}}$$ ($\pi^2 \approx 10$)

For n = 0, V_C = 10 V and for n = 10000, V_C = 3.015 V

The plot is shown in figure.



(c) Plot V_R versus frequency for the frequency range of part (a).

As above in part b, the current in the circuit is given by

$$I_{eff} = \frac{V_{eff}}{Z} = \frac{10}{\sqrt{100^2+\left(\frac{1}{C\omega}\right)^2}}$$

The voltage across the resistor R is given by

$$V_R = I_{eff} * R = \frac{10R}{\sqrt{100^2+\left(\frac{1}{C\omega}\right)^2}}$$

Or $$V_R = \frac{10*100}{\sqrt{10^4+\left(\frac{1}{C\omega}\right)^2}} = \frac{1000}{\sqrt{10^4+\left(\frac{1}{C*2\pi n}\right)^2}} = \frac{10}{\sqrt{1+\frac{10^7}{n^2}}}$$

Or $$V_R = \frac{10}{\sqrt{1+\frac{10^7}{n^2}}} \qquad (\pi^2 \approx 10)$$

For n = 0, V_R = 0 and for n = 10000, V_R = 9.53 V

The plot is shown in the figure bellow