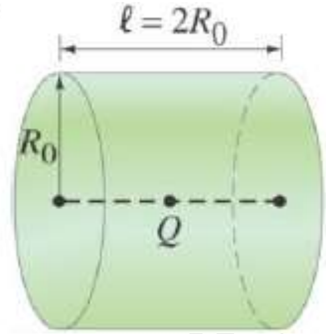
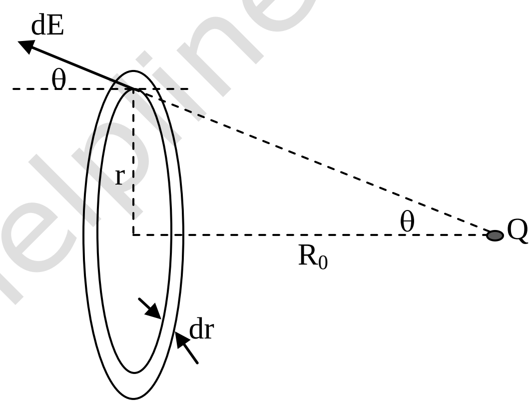


Q. A point charge Q is on the axis of a short cylinder at its center. The diameter of the cylinder is equal to its length L (Fig.). What is the total flux through the curved sides of the cylinder?

The flux coming out of a point charge Q is $\frac{Q}{\epsilon_0}$ and uniformly distributed in all directions. Thus the total flux through the whole surface of the cylinder is $\frac{Q}{\epsilon_0}$

Now to calculate the flux through a plane surface of the cylinder, we may consider it to be formed by number of thin concentric rings. Consider a thin ring of radius r and width dr as in diagram. The distance of any point on the ring from the charge Q will be given by $\sqrt{R_0^2 + r^2}$ and hence the magnitude of the infinitesimally small field dE at ring is given by

$$dE = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R_0^2 + r^2}\right)$$



The component of this field parallel to the axis of cylinder and normal to the plane surface of the cylinder is given by

$$dE_x = dE * \cos\theta$$

Or $$dE_x = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{R_0^2 + r^2}\right) * \frac{R_0}{\sqrt{R_0^2 + r^2}}$$

Or $$dE_x = \frac{1}{4\pi\epsilon_0}\left[\frac{QR_0}{(R_0^2 + r^2)^{\frac{3}{2}}}\right]$$

Hence the flux through this ring surface is given by the product of the component of the field normal to the surface and surface area of the ring, thus

$$d\emptyset = dE_x * 2\pi r\, dr = \frac{1}{4\pi\epsilon_0}\left[\frac{QR_0}{(R_0^2 + r^2)^{\frac{3}{2}}}\right] * 2\pi r\, dr$$

Or $$d\emptyset = \frac{1}{2\epsilon_0}\left[\frac{QR_0\, r\, dr}{(R_0^2 + r^2)^{\frac{3}{2}}}\right]$$

Thus the total flux through the end plane surface of the cylinder is given by integrating above equation with proper limits as

$$\emptyset = \int d\emptyset = \frac{QR_0}{2\epsilon_0}\int_0^{R_0} \frac{r\, dr}{(R_0^2 + r^2)^{\frac{3}{2}}}$$

Now substituting $R_0^2 + r^2 = u^2$

Differentiating we get 2 r dr = 2 u du

Or r dr = u du

For r = 0; u = R_0 and for r = R_0; u = $\sqrt{2}\, R_0$

Thus the equation above reduces to

$$\emptyset = \frac{QR_0}{2\epsilon_0}\int_{R_0}^{\sqrt{2}\,R_0} \frac{u\,du}{u^3}$$

Or $$\emptyset = \frac{QR_0}{2\epsilon_0}\int_{R_0}^{\sqrt{2}\,R_0} \frac{du}{u^2}$$

Or $$\emptyset = \frac{QR_0}{2\epsilon_0}\left[-\frac{1}{u}\right]_{R_0}^{\sqrt{2}\,R_0}$$

Gives $$\emptyset = \frac{QR_0}{2\epsilon_0}\left[\frac{1}{R_0} - \frac{1}{\sqrt{2}\,R_0}\right]$$

Or $$\emptyset = \frac{(\sqrt{2}-1)Q}{2\sqrt{2}\,\epsilon_0}$$

Now as there are two such plane surfaces and hence the flux through the curved surface is given by

$$\emptyset_{curved} = \frac{Q}{\epsilon_0} - 2\emptyset$$

Or $$\emptyset_{curved} = \frac{Q}{\epsilon_0} - 2\frac{(\sqrt{2}-1)Q}{2\sqrt{2}\,\epsilon_0}$$

Or $$\emptyset_{curved} = \frac{Q}{\epsilon_0}\left(1 - \frac{(\sqrt{2}-1)}{\sqrt{2}}\right)$$

Or $$\emptyset_{curved} = \frac{Q}{\sqrt{2}\,\epsilon_0}$$