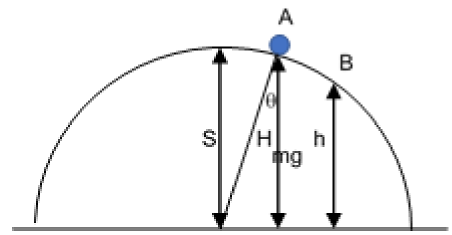
Q- A solid sphere of radius R starts at rest on top of a semicircular track of radius S., as shown in the figure. The sphere rolls *without slipping* along the track until it falls off the track at point B. Point A is a vertical distance H above the ground; the height of point B is unknown. You may assume that R<<S.




When a solid sphere of mass m and radius R rolls without slipping on a surface with velocity v, as the velocity of centre of mass is v, its translational kinetic energy is given by

$$E_{Trans} = \frac{1}{2}\, mv^2$$

The moment of inertia of the sphere about its diameter is (2/5) $mR^2$.

As there is no slipping, the point of contact has zero velocity and hence the angular velocity of the sphere about its centre must be $\omega = v/R$.

Thus, the rotational kinetic energy of the sphere is given by

$$E_{Rotation} = \frac{1}{2}\, I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$$

Gives $E_{Rotation} = \frac{1}{5}mv^2$

Hence the total kinetic energy stored in a solid sphere of mass m rolling without slipping on a surface, with velocity v is given by

$$E_{Total} = E_{Trans} + E_{Rotational} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2 = \frac{7}{10}m\omega^2R^2$$

(a) What is the angular speed $\omega_A$ of the sphere at point A, expressed as a function of H, R, S, and the (downward) acceleration of gravity *g*?

When the sphere roll without slipping, no work is done against friction and hence according to law of conservation of mechanical energy we have

Gain in kinetic energy = loss in potential energy

Here initial kinetic energy of the sphere is zero. If the angular velocity of the sphere at point A is □□, its total kinetic energy will be

$$E_A = \frac{7}{10}m{\omega_A}^2R^2$$

The loss in potential energy as the sphere rolls from the top at height S to point A at height H is given by

$$\Delta U = m\, g\, (S - H)$$

And hence according to law of conservation of energy we get

` $$\frac{7}{10}m{\omega_A}^2R^2 = mg\,(S - H)$$

Or $${\omega_A}^2 = \frac{10g\,(S-H)}{7\,R^2}$$

Or $$\omega_A = \sqrt{\frac{10g\,(S-H)}{7\,R^2}}$$ ----------------------- (1)

Now we want to solve for the height *h* at which the sphere falls off the track. Proceed as follows:

(b) Find the speed of the sphere at point B, where it falls off the track.

As the centre of mass of the sphere is moving on a circular track, it requires a centripetal force towards the centre of the track. This force is provided by the radial component of the weight of the sphere. The normal reaction adjusts the extra force such that

$$mg\cos\theta - N = \frac{mv^2}{s}$$

The tangential component of the force will accelerate the sphere.

As the sphere comes down, the velocity and hence the required centripetal force both increases, but the radial component of its weight decreases. Thus, the normal reaction decreases and at some point (B) becomes zero. When the normal reaction N just becomes zero the contact between the sphere and the track vanishes and the sphere leaves the track. This happens at point B and hence for point B we can write above equation as

$$mg\cos\theta = \frac{mv^2}{S}$$

Or $v^2 = gS\cos\theta$

Gives $v = \sqrt{gS\cos\theta}$

As in triangle OBC the radius OB = S and the height BC =h we get cos θ = h/S and hence the speed of the sphere at point B to leave the track is given by

$$v = \sqrt{\frac{gSh}{S}} = \sqrt{gh}$$

Hence the speed of the rolling sphere at B must be equal to

$$v = \sqrt{\boldsymbol{gh}} \qquad \text{-----------------------} \ (2)$$