Q- Two masses 5 kg and 15 kg are moving along $x$ direction with velocities of $10 \mathrm{~m} / \mathrm{s}$ and $2 \mathrm{~m} / \mathrm{s}$ respectively. At an instant they are at position 10 m and 70 m respectively
(a) Find position of their center of mass at this instant.
(b) If the two will collide inelastically what is their common velocity?
(c) If they collide elastically find the velocity of each one after collision.

a) The position of center of mass of a system of particles of mass $m_{1}, m_{2}, m_{3} \ldots$ at positions $r_{1}, r_{2}, r_{3} \ldots$. respectively is given by

$$
\vec{r}_{c m}=\frac{m_{1} \vec{r}_{1}+m_{2} \vec{r}_{2}+m_{3} \vec{r}_{3}+\cdots}{m_{1}+m_{2}+m_{3}+\cdots}
$$

Here we have only two objects and both objects are on $x$ axis ( $y$ and $z$ coordinates are zero) hence the $x$ coordinate of their center of mass is given by

$$
\begin{aligned}
x_{c m} & =\frac{m_{1} x_{1}+m_{2} x_{2}}{m_{1}+m_{2}} \\
\text { Or } \quad x_{c m} & =\frac{5 * 10+15 * 70}{5+15}=55 \mathrm{~m}
\end{aligned}
$$

Hence the center of mass of the system of the two bodies will be at 55 m from origin.

## Go through the reading Head-on Collision (Home page)

b) $\mathrm{m}_{1}=5 \mathrm{~kg} ; \quad \mathrm{u}_{1}=10 \mathrm{~m} / \mathrm{s} ; \mathrm{m}_{2}=15 \mathrm{~kg} ; \quad \mathrm{u}_{2}=2 \mathrm{~m} / \mathrm{s}$;

As after collision the bodies sticks together, they move with a common velocity v.
As there is no external force on the system, according to law of conservation of linear momentum we have Momentum of the system after collision = momentum before collision
Or $\quad m_{1} v+m_{2} v=m_{1} u_{1}+m_{2} u_{2}$
Gives $v=\frac{m_{1} u_{1}+m_{2} u_{2}}{m_{1}+m_{2}}=\frac{5 * 10+15 * 2}{5+15}=\frac{80}{20}=4 \mathrm{~m} / \mathrm{s}$
Hence their common velocity after collision will be $4 \mathrm{~m} / \mathrm{s}$
c) If the collision is elastic

According to law of conservation of linear momentum

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    m}\mp@subsup{m}{1}{}\mp@subsup{v}{1}{}+\mp@subsup{m}{2}{}\mp@subsup{v}{2}{}=\mp@subsup{m}{1}{}\mp@subsup{u}{1}{}+\mp@subsup{m}{2}{}\mp@subsup{u}{2}{
or
5*}\mp@subsup{\textrm{v}}{1}{}+1\mp@subsup{5}{}{*}\mp@subsup{\textrm{v}}{2}{}=\mp@subsup{5}{}{*}10+15*
or }\quad\mp@subsup{\mathbf{v}}{\mathbf{1}}{}+\mathbf{3}\mp@subsup{\mathbf{v}}{\mathbf{2}}{\mathbf{=}}=\mathbf{16
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And for elastic collision the velocity of separation is equal to velocity of approach ( $\mathrm{e}=0$ ) hence we have
$\mathrm{v}_{2}-\mathrm{v}_{1}=\mathrm{u}_{1}-\mathrm{u}_{2}$
Or $\quad v_{2}-v_{1}=10-2$
Or
$\mathbf{v}_{\mathbf{2}}-\mathbf{v}_{\mathbf{1}}=\mathbf{8}$
Adding the two equations we get

$$
4 v_{2}=24
$$

Or $\quad \mathrm{v}_{2}=6 \mathrm{~m} / \mathrm{s}$
Substituting value of $\mathrm{v}_{2}$ in equation (1) we get
$\mathrm{V}_{1}+3^{*} 6=16$
Or $\quad \mathrm{v}_{1}=-2 \mathrm{~m} / \mathrm{s}$
Here negative sign shows that the first body will move in opposite direction after collision.
Hence after collision first body will move with velocity - $\mathbf{2} \mathbf{~ m} / \mathrm{s}$
and the second body with velocity of $6 \mathrm{~m} / \mathrm{s}$.

