Q- Using mesh analysis, determine the current through $R_3 = 5 \Omega$ resistance and the potential V_a .

Let the current in R_1 is I_1 , R_2 is I_2 and R_3 is I_3 . The currents other resistors are indicated in the figure using the junction law.

Using the loop law for mesh dbcd we get

$$\Sigma E = \Sigma IR$$

Or
$$E_1 = I_1 R_1 + (I_1 - I_3) R_4$$

Or
$$12 = 4 I_1 + 6*(I_1 - I_3)$$

Or
$$10 I_1 - 6 I_3 = 12$$

Or
$$5 I_1 - 3 I_3 = 6$$
 ----- (1)

Using the loop law for mesh dbad we get

$$\Sigma E = \Sigma IR$$

Or
$$E_1 = I_1R_1 + I_3R_3 + (I_3 - I_2)*R_5$$

Or
$$12 = 4 I_1 + 5 I_3 + 3*(I_3 - I_2)$$

Or
$$4 I_1 - 3 I_2 + 8 I_3 = 12$$
 ----- (2)

And for mesh acda we have

$$\Sigma E = \Sigma IR$$

Or
$$E_2 = I_2 R_2 - (I_3 - I_2) R_5$$

Or
$$16 = 2 I_2 - 3*(I_3 - I_2)$$

Or
$$5 I_2 - 3 I_3 = 16$$
 ----- (

Substituting I_1 from (1) and I_3 from (3) in equation (2) we get

$$4\left(\frac{6+3I_3}{5}\right) - 3I_2 + 8\left(\frac{5I_2-16}{3}\right) = 12$$

Or
$$72 + 36 I_3 - 45 I_2 + 200 I_2 - 640 = 180$$

Or
$$155 I_2 + 36 I_3 = 748$$
 ----- (4

Solving equations (4) + (3)*12 we get

$$215 I_2 = 940$$

Or
$$I_2 = 940/215 = 4.372 A$$

Substituting in equation (3) we get

$$I_3 = (5*4.372 - 16)/3 = 1.953 A$$

Hence potential at b, Va is given by

$$Va = I_2R_2 - E_2 = 4.372*2 - 16 = -7.356 V$$

