

Q- A rocket is fired at an angle of 60° with horizontal and accelerates at 25m.s^{-2} for 2.5s before the rocket motor stops and it eventually falls to the ground. Neglecting air resistance and assuming that the trajectory during acceleration is a straight line.

(a) Make a labelled sketch of the rockets trajectory starting from the launch point. Include the numerical values of the horizontal and vertical velocity at the point where the rocket motor stops.

(b) How long after launch does the rocket hit the ground?

(C) How far does the rocket travel horizontally from the launch point?

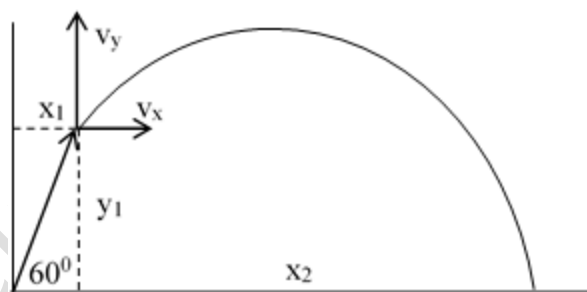
The rocket is moving in a vertical plane under gravity. The motion is known as projectile motion and can be solved directly using formulae or by solving motion in horizontal and vertical direction separately.

(a) We have to assume that the rocket is moving in a straight line during its launch, means the gravity is to be neglected for this period and the velocity is increasing at a constant rate of 25 m/s^2 . its velocity after 2.5 s can be calculated using the equation of motion

$$[v = u + a \cdot \Delta t]$$

$$v = 0 + 25 \cdot 2.5$$

gives $v = 62.5\text{ m/s}$



As this velocity is at an angle of 60° to the horizontal resolving this, we get the horizontal component v_x and the vertical component v_y of the it as

$$v_x = v \cdot \cos \theta = 62.5 \cos 60^\circ = 62.5 \cdot 0.5 = 31.25\text{ m/s}$$

and $v_y = v \cdot \sin \theta = 62.5 \sin 60^\circ = 62.5 \cdot 0.866 = 54.13\text{ m/s}$

Similarly, the distance covered during this period is given by using the equation

$$[s = u \cdot t + (1/2) a \cdot \Delta t^2]$$

Or $s = 0 + 0.5 \cdot 25 \cdot (2.5)^2 = 78.13\text{ m}$.

Again, the horizontal and vertical distances from the point of launching is given respectively by resolving s , as

$$x_1 = s \cos \theta = 78.13 \cos 60^\circ = 39.07\text{ m}$$

And $y_1 = s \sin \theta = 78.13 \sin 60^\circ = 67.66\text{ m}$

(b) When the motor stops the rocket is at a height of $y_1 = 67.66\text{ m}$ and moving with vertical velocity (upward) $v_y = 54.13\text{ m/s}$. Let the time from this point to the ground is t seconds. During this time the vertical displacement is $-y_1$ and the acceleration is that due to gravity $g = -9.8\text{ m/s}^2$. Hence for the vertical motion

$$[s = u \cdot t + (1/2) a \cdot \Delta t^2]$$

$$-67.66 = 54.13 \cdot \Delta t + 0.5 \cdot (-9.8) \Delta t^2$$

or $4.9 \cdot \Delta t^2 - 54.13 \cdot \Delta t - 67.66 = 0$

$$\text{or } \Delta t = \frac{54.13 \pm \sqrt{(-54.13)^2 - 4 \cdot 4.9 \cdot (-67.66)}}{2 \cdot 4.9} = \frac{54.13 \pm 65.24}{9.8} = 12.2 \text{ s}$$

(Time cannot be negative)

Hence the total time taken to reach ground is $2.5 + 12.2 = 14.7$ seconds.

(c) During the curvilinear motion for time t , the rocket is moving horizontally with constant horizontal velocity v_x and hence the horizontal distance covered is given by

$$x_2 = v_x \cdot \Delta t = 31.25 \cdot 12.2 = 381.3 \text{ m}$$

Hence the total horizontal distance from the point of launch

$$x = x_1 + x_2 = 39.06 + 381.3 = 420.3 \text{ m.}$$
