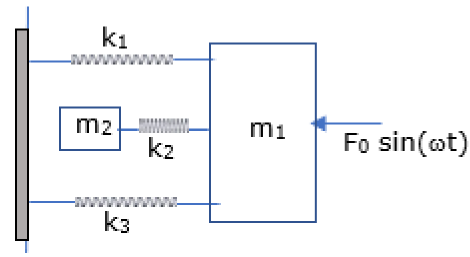
Q- A harmonic force [$F_0 \sin(\omega t)$] is applied to mass m_1 which in turn is coupled to springs of spring constants k_1, k_2 and k_3. Let x_1 and x_2 be the deviations from equilibrium for m_1 and m_2, respectively. Ignore gravity. The blocks move on a horizontal frictionless table.

(a) Write the equations of motion. Use x_1 and x_2 as your variables.

(b) Find the normal frequencies of the system, that is, when the external force is absent. Assume for this part and the remainder of the problem the springs are the same and the masses are equal.

(c) Solve for the steady state motion with the force, that is find x_1 and x_2 as functions of time.



In equilibrium position the springs are having natural lengths and let at time t = o the external force $F_0 \sin \omega t$ is applied to m_1. Let at time t the displacement of m_1 is x_1 and that of m_2 is x_2. (Left is positive)

(a) Write the equations of motion. Use x_1 and x_2 as your variables.

At time t, compression in spring 1 and 3 will be x_1 and in spring 2 will be $x_1 - x_2$. Hence the tension in springs 1, $T_1 = K_1 x_1$; in spring 2, $T_2 = K_2 (x_1 - x_2)$; and in spring 3, $T_3 = K_3 x_1$.

Hence the equation of motion for m_1 is given by

$$m_1 \frac{d^2 x_1}{dt^2} = F_0 \sin \omega t - K_1 x_1 - K_3 x_1 - K_2 (x_1 - x_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 1$$

And equation for m_2 is

$$m_2 \frac{d^2 x_2}{dt^2} = K_2 (x_1 - x_2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.$$

(b) Find the normal frequencies of the system, that is, when the external force is absent. Assume for this part and the **remainder of the problem** the springs are the same and the masses are equal.

According to the question, now for the remaining problem we have to take the masses and the springs same. Let $m_1 = m_2 = m$ and $K_1 = K_2 = K_3 = K$.

The external force is zero.

Now for sustend oscillations both the blocks must have **same frequency**.

There will be two different ways

One the two block moves always in the same direction, and the other in which the two moves always in opposite directions.

In the first case if we push the block 2 towards block 1 slowly up to force F_0, to the extreme position then release from rest in this the situation the restoring forces on the two are given by

$$F_2 = - K (x_2 - x_1) \text{ and } F_1 = - 2Kx_1 + K(x_2 - x_1)$$

As the frequencies and the mass of both are equal the restoring forces must proportional to the displacements and hence we get

$$x_1(x_2 - x_1) = x_2 [2x_1 - (x_2 - x_1)]$$

gives $x_2 = (\pm\sqrt{2} + 1)x_1$

So the oscillation will be such that always $x_2 = (\pm\sqrt{2} + 1)x_1$

Hence the equations of motion for both blocks reduces to

$$m\frac{d^2x_1}{dt^2} = - K(2 \pm \sqrt{2})\, x_1 \quad \text{and} \quad m\frac{d^2x_2}{dt^2} = - K(2 \pm \sqrt{2})\, x_2$$

hence the frequencies of oscillation are given by

$$n = \frac{1}{2\pi}\sqrt{\frac{(2 \pm \sqrt{2})K}{m}}$$

The sign for this situation is positive, the negative sign is corresponding to the situation in which the blocks always moving opposite direction.

Bringing the blocks nearer and then releasing them can solve this situation in the same manner.

3) The displacement of the a body executing simple harmonic motion is given by

$x = A \cos \omega t$ when the particle is released from the extreme position.

Hence the displacement of block 1 is given by { as $\omega = 2\pi n$

$$x_1 = A \cos 2\pi n = A \cos\sqrt{\frac{(2 \pm \sqrt{2})K}{m}}$$

and $x_2 = (\pm\sqrt{2} + 1)x_1 = (\pm\sqrt{2} + 1) A \cos\sqrt{\frac{(2 \pm \sqrt{2})K}{m}}$