Q- A circular steel plate of radius 0.10 m is cooled from 350^0C to 20^0C. By what percentage does the plate's area decrease?
(Coefficient of linear expansion of steel α = $1.2*10^{-5}$ /^{0}C)

With cooling the radius of the plate decreases linearly and hence the change in the radius is given by

$$\Delta R = \alpha * R * \Delta T$$

Radius of the plate R = 0.10 m

Increase in temperature ΔT = 20 - 350 = - 330^0C

Coefficient of linear expansion of steel α = $1.2*10^{-5}$ /^{0}C

Thus $\Delta R = 1.2 * 10^{-5} * 0.10 * (-330) = -3.96 * 10^{-4} m$ ---------- (1)

Now the initial area of the plate

$$A = \pi R^2$$

With decrease in radius the area becomes

$$A' = \pi (R - \Delta R)^2$$

Thus, decrease in area will be given by

$$A - A' = \pi [R^2 - (R - \Delta R)^2]$$

Or $$\Delta A = \pi [R^2 - (R^2 - 2R * \Delta R + \Delta R^2)]$$

Or $$\Delta A = \pi [R^2 - R^2 + 2R * \Delta R - \Delta R^2]$$

Or $$\Delta A = 2\pi R * \Delta R$$

[as ΔR << R; ΔR^2 ≪≪ R^2 and hence neglecting as compared to other quantities]

Thus the % change in the area of the plate will be given by

$$p = \frac{\Delta A}{A} * 100\ \% = \frac{2\pi R * \Delta R}{\pi R^2}\% = \frac{2 * \Delta R}{R}\%$$

Substituting the values, we get

$$p = \frac{2 * 3.96 * 10^{-4}}{0.10} = 7.92 * 10^{-3}\%$$

Hence the percentage change in the area of the plate will be $7.92*10^{-3}$ %