Q- A mass in the form of a solid cylinder, of radius c acted upon by no forces, moves parallel to its axis through a uniform cloud of fine dust, of volume density ρ, which is at rest. If the particles of dust which meet the mass adhere to it, and if M and u be the mass and the velocity at the beginning of the motion, prove that the distance x traversed in time t is given by the equation

$$(M + \rho\pi c^2 x)^2 = M^2 + 2\rho\pi u c^2 M t \quad .$$

Mass of the system moving at any time t is the mass of the cylinder and the mass of the dust deposited on the front plane surface of the cylinder, therefore this is the case of variable mass system and the mass of the system as a function of time is given by

$$m = M + \pi . c^2 x \rho$$

Here x is the distance traversed by the cylinder in time t.

If at time t, in an infinitesimal time dt it covers a distance dx (velocity dx/dt), according to law of conservation of linear momentum (as there is no any external force on the system)

$$Mu = m\frac{dx}{dt}$$

Or $$Mu = (M + \pi c^2 x\rho)\frac{dx}{dt}$$

Or $$(M + \pi c^2 x\rho)\, dx = Mu\, dt$$

Integrating

$$\int_0^x (M + \pi c^2 x\rho)\, dx = \int_0^t Mu\, dt$$

Or $$Mx + \frac{1}{2}\pi c^2 \rho x^2 = Mu\, t$$

Or $$2Mx + \pi c^2 \rho x^2 = 2Mu\, t$$

Or $$(2Mx + \pi c^2 \rho x^2)\pi c^2 \rho = 2Mut.\pi c^2 \rho$$ (multiplying each side by $\pi c^2 \rho$)

Or $$M^2 + (2Mx + \pi c^2 \rho x^2)\pi c^2 \rho = M^2 + 2Mut.\pi c^2 \rho$$ (adding M^2 to each side)

Or $$M^2 + 2M.\pi c^2 \rho . x + \pi^2 c^4 \rho^2 x^2 = M^2 + 2Mut.\pi c^2 \rho$$

Or $$(M + \rho\pi c^2 x)^2 = M^2 + 2\rho\pi u c^2 M t$$

This is the required equation.