Q- A roller coaster car of mass 1000 kg is riding along a track from point A to point C as in figure. ($g=10 m/s^2$)

(a) Assuming no friction, what is the speed of the car at the top of first hill A?

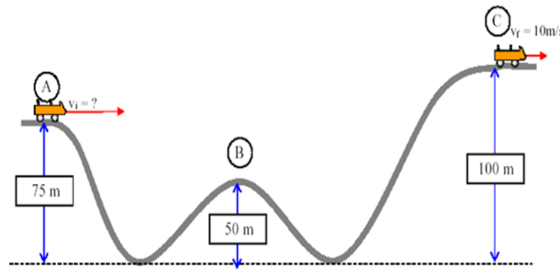
As there are no non-conservative forces the mechanical energy of the system remains conserved and hence

initial energy = final energy

or initial kinetic energy + initial potential energy = final kinetic energy + final potential energy

or $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$

or $v_i^2 + 2gh_i = v_f^2 + 2gh_f$



Substituting the values, we get

$v_i^2 + 2*10*75 = 10^2 + 2*10*100$

Gives $v_i^2 = 600$

Or $v_i = \sqrt{600} = 24.49$ m/s

(b) What is the speed of the car at the top of second hill B?

Let the speed of the car at the top of the second hill is v then applying the law of conservation of energy for points A and B we have

initial kinetic energy + initial potential energy = final kinetic energy + final potential energy

or $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv^2 + mgh$

or $v_i^2 + 2gh_i = v^2 + 2gh$

or $600 + 2*10*75 = v^2 + 2*10*50$

or $v^2 = 600 + 1500 - 1000 = 1100$

gives v = 33.17 m/s

(c) What is the maximum speed of the car along the track?

The maximum speed car can achieve will be at the lowest point of the track where the whole energy will be converted in kinetic energy hence the speed at the lowest point and according to the law of conservation of energy at A and the lowest point of the track is given by

$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_{max}{}^2$

or $v_i^2 + 2gh_i = v_{max}^2$

or $600 + 2*10*75 = v_{max}2$

or $v_{max}2 = 600 + 1500 = 2100$

Gives $v_{max} = 45.83$ m/s

(d) Assume there is friction and the car start with the same speed at A. If the work done against friction from A to C is 50000 J, will the car still reach C and if so what is its velocity at C?

Total energy at A = $\frac{1}{2}mv_i^2 + mgh_i = \frac{m}{2}\left(v_i^2 + 2gh_i\right) = 500\left(600 + 2*10*75\right) = 1050000$ J

The work done against friction is 50000 J hence the energy balance if it reaches point C will be
1050000 - 50000 = 1000000 J

Now at point C the potential energy of the car will be

$$mgh_f = 1000*10*100 = 1000000 \text{ J}$$

Which is just equal to the energy balance and hence the kinetic energy at this point will be zero hence the car will just reach the top of the third hill and the speed there will be just zero.