Q- Air enters the compressor of a gas turbine plant (operating at steady state) at ambient conditions of 1 bar, 25^0C with low velocity and exits at 10 bar, 350^0C with a velocity of 90 m/s. The compressor is cooled at a rate of 1500 kJ/min and the power input to the device is 250 kW. Determine the mass flow rate of air through the compressor. Also determine the area A of the exit duct.

At input

The pressure of air $P_1 = 1 \text{ bar} = 10^5$ Pa

Temperature $T_1 = 25+273 = 298$ K

Rate of heat taken out in cooling $r_1 = 1500$ KJ/min = 1500/60 = 25 KW

Power input $r_2 = 250$ KW

At exit

The pressure of air $P_2 = 10 \text{ bar} = 10^6$ Pa

Temperature $T_2 = 350 + 273 = 623$ K

Velocity of the air at exit $v = 90$ m/s

As the type of process is not given it will be easy to take energy considerations.

If we consider a controlled volume of air in the compressor, and the steady state then the total change of internal energy inside the compressor will be zero and we can write

$$H_{in} - H_{out} + W_{in} - Q_{out} - KE = 0 \quad \text{-----------------} \ (1)$$

Here

H_{in} = Enthalpy (or heat content) entering the compressor with input air.

H_{out}= Enthalpy coming out of the compressor with output air.

W_{in}= Work going in as the power input.

Q_{out} = Heat being given out in cooling, and

KE = Kinetic energy acquired by air.

Let m **kg** of the air flows through the compressor per second.

As the enthalpy is given by

$$H = U + PV = n*Cv*T + n\,RT \qquad [PV = n\,RT]$$

Or $$H = n\,(Cv + R)\,T = n\,Cp\,T$$

Here Cv and Cp are the molar specific heats of air at constant volume and constant pressure respectively

Thus the input and output enthalpy is given by

H_{in} = m Cp T_1 = m (kgS^{-1})*1.012($kJ.kg^{-1}.K^{-1}$)*298K = m*301.6 KW

H_{out} = m Cp T_2 = m(kgS^{-1})*1.012($kJ.kg^{-1}.K^{-1}$)*623 K = m*630.5 KW

And the kinetic energy of outgoing air will be

KE = ½ mv^2 = 0.5*m*90^2 = m*4050 W = m*4.05 KW

Given Win = 250 KW

And Qout = 25 KW

Substituting all data in equation (1) we get

H_{in} - H_{out} + W_{in} – Q_{out} – KE = 0

Or m*301.6 – m*630.5 + 250 – 25 – m*4.05 = 0

Gives $m = \frac{250-25}{4.05+630.5-301.6} = \frac{225}{332.95} = 0.676\ Kg/s$

Volume of air coming out per second will be given by

PV = nRT

Or $V = n\frac{RT}{P} = \left(\frac{1000m}{M}\right)\frac{RT}{P} = \left(\frac{1000*0.676}{29}\right)\frac{8.31*623}{10^6}$

Or V = 0.121 m^3

Now as the exit velocity of air is v = 90 m/s the area A of the exit duct is given by

V = A*v

Or A = V/v = 0.121/90 = $1.34*10^{-3}$ m^2.