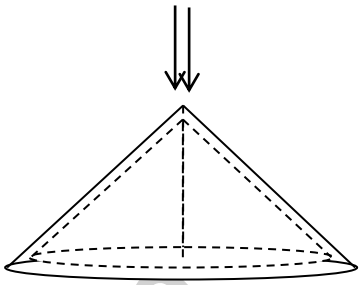
Q- At a sand and gravel plant, sand is falling off a conveyor and onto a conical pile at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately 3x the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?

Solution:



As more and more sand falling the height of the pile and the radius of the base increases. Let at some time t the height of the pile is y and thus according the question the diameter of the base is 3y. volume of the sand in the pile is given as volume of the cone

$V = [(1/3)\pi R^2 h] = (1/3)\ \pi\ (3y/2)^2\ y = (3\ \pi\ y3)/4$

The rate of change of the volume of the pile is given by the differential coefficient of the volume V with respect to the time and hence

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{3\pi y^3}{4}\right) = \frac{3\pi}{4}\frac{d}{dt}\left(y^3\right) = \frac{3\pi}{4}\frac{d}{dy}\left(y^3\right)\frac{dy}{dt} = \frac{3\pi}{4}\left(3y^2\right)\frac{dy}{dt} = \frac{9\pi y^2}{4}\left(\frac{dy}{dt}\right)$$

gives

$$\frac{dy}{dt} = \frac{4}{9\pi y^2}\frac{dV}{dt}$$

Here rate of increase of volume dV/dt is the rate of falling of sand = 10 cubic feet per min. hence substituting the values we get, the rate of increase in height dy/dt when the height of the pile is 15 feet as--

$$\frac{dy}{dt} = \frac{4\times 10}{9\times\pi\times 225} = \frac{8}{405\pi}$$ foot per min.

This is the required answer.