Q- A motor boat, with its engine on in a running river and blown over by a horizontal wind is observed to travel at 20 km/hr in a direction 53^0 East of North. The velocity of the boat with its engine on in still water and blown over by the horizontal wind is 4 km/hr Eastward and the velocity of the boat with its engine on over the running river, in the absence of wind is 8 km/hr due South. Find (a) the velocity of the boat in magnitude and direction, over still water in the absence of wind (b) the velocity of the wind in magnitude and direction.

Let the velocity of boat in still water in absence of wind be $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$. Velocity of river flow is $\vec{v}_R = v_{Rx}\hat{\imath} + v_{Ry}\hat{\jmath}$ and velocity of wind is $\vec{v}_w = v_{wx}\hat{\imath} + v_{wy}\hat{\jmath}$.

The velocity of boat with engine on and wind and river flow is 20 m/s, 53^0 east of north gives us

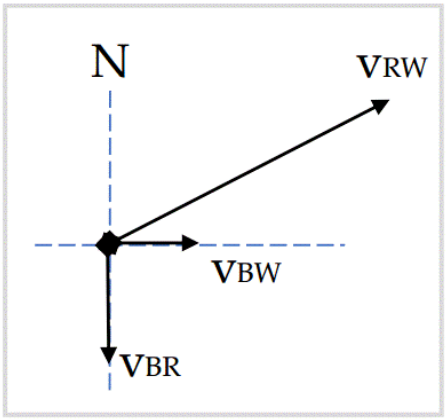
$$\vec{v}_{RW} = \vec{v} + \vec{v}_R + \vec{v}_w = 20\sin 53^0\,\hat{\imath} + 20\cos 53^0\,\hat{\jmath}$$

Or $\vec{v} + \vec{v}_R + \vec{v}_w = 20 * \frac{4}{5}\,\hat{\imath} + 20 * \frac{3}{5}\,\hat{\jmath}$

Or $\vec{v} + \vec{v}_R + \vec{v}_w = 16\,\hat{\imath} + 12\,\hat{\jmath}$ --- (1)

And $\vec{v}_{BW} = \vec{v} + \vec{v}_w = 4\,\hat{\imath}$ --- (2)

and $\vec{v}_{BR} = \vec{v} + \vec{v}_R = -8\,\hat{\jmath}$ --- (3)



Subtracting equations 3 3 from equation 1 we get velocity of the boat in still water as

$$-\vec{v} = 16\,\hat{\imath} + 12\,\hat{\jmath} - 4\,\hat{\imath} - (-8\,\hat{\jmath})$$

Or $\vec{v} = -12\,\hat{\imath} - 20\,\hat{\jmath}$

And its speed (magnitude of velocity)

$$v = \sqrt{(-12)^2 + (-20)^2} = 23.3\ km/hr$$

in the direction $\theta = \tan^{-1}\frac{20}{12} = \tan^{-1} 1.66 = 59^0$ South of west.

(both components are negative, third quadrant)

Substituting for v in equation 2 we get

$$\vec{v}_{BW} = \vec{v} + \vec{v}_w = 4\,\hat{\imath}$$

Or $\vec{v}_w = 4\,\hat{\imath} - \vec{v} = 4\,\hat{\imath} + 12\,\hat{\imath} + 20\,\hat{\jmath}$

Or $\vec{v}_w + 16\,\hat{\imath} + 20\,\hat{\jmath}$

or magnitude $v_w = \sqrt{(16)^2 + 20^2} = 25.6\ km/hr$

in the direction $\theta = \tan^{-1}\frac{20}{16} = \tan^{-1} 1.25 = 51.3^0$ North of east.