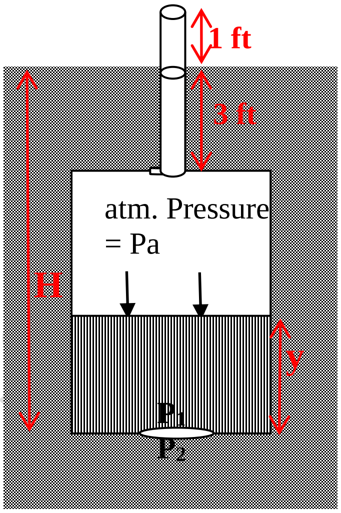
Q- A container full of air at atmospheric pressure is Three feet below the surface in a body of water; this container is 8 feet high by 3 feet square. A 16-inch ID valve is on the bottom of the container and a 2-inch valve is on the top. A 2-inch ID hose connects to this valve and extends 1 foot above the surface of the water. Both valves are opened at the same time; calculate the length of time required for the container to fill with water.

Given:



3ft x 3ft x 8ft are dimensions of the tank,
h = 8 ft, g = 32.17 ft/s^2 is acceleration due to gravity (in FPS)

Let y be the height of water in the tank above the bottom valve at time t.

The speed of water through the tank as function of time t is given by

$v_2 = dy/dt$

Writing the Bernoulli equation for the water, just below and just above the lower valve where the pressures are P_1 and P_2 respectively. (Considering that the velocity of water outside is zero due to large area and inside is v_1). This gives the velocity of the flow of water in to the tank as

$$P_1 + \frac{1}{2}\rho\, v_1^2 = P_2 + \frac{1}{2}\rho\,(0)^2$$

Or $$y\rho g + \frac{1}{2}\rho\, v_1^2 = H\rho g + 0$$

Gives $g*(H - y) = v_1^2/2$

From which we get

$$v_1 = \sqrt{2g(H-y)}$$

As the area of the valve and the tank are different thus the velocity with which water rises in the tank v_2 is given by using equation of continuity as

$A_1v_1 = A_2v_2$

And hence the velocity of the rise of water level in the tank is given by

$$v_2 = \frac{A_1 v_1}{A_2}$$

Or $$\frac{dy}{dt} = \frac{A_1\sqrt{2g(H-y)}}{A_2}$$

Substitute v and get the differential equation

$$\frac{dy}{dt} = \frac{A_1\sqrt{2g(H-y)}}{A_2}$$

Separate variables and get

$$\frac{dy}{\sqrt{2g(H-y)}} = \frac{A_1}{A_2}\,dt$$

Integrate both parts until the water fills the tank

$$\int_0^h \frac{dy}{\sqrt{2g(H-y)}} = \frac{A_1}{A_2}\int_0^t dt$$

Integrating gives the length of time required for the container to be filled with water considering that the reservoir is large enough, so H remains constant, and H = 3 + h = 3 + 8 = 11 ft, we get

$$\frac{A_1}{A_2}\,t = \sqrt{\frac{1}{2g}}\;(-2)(\sqrt{H-h} - \sqrt{H})$$

Or $\quad t = \frac{A_2}{A_1}\sqrt{\frac{2}{g}}(\sqrt{H} - \sqrt{H-h})$

Putting the numbers gives the answer

$t = \frac{3*3}{\pi R^2}\sqrt{\frac{2}{g}}(\sqrt{H} - \sqrt{H-h})$ (R is the inner radius = 2/2 = 1 inch = 1/12 ft.

or $\quad t = \frac{3*3*12*12}{3.14}\sqrt{\frac{2}{32.17}}(\sqrt{11} - \sqrt{11-3})$

or $\quad t = 412.7 * 0.249(3.317 - 2.828) = 50.2\ s$