Q- In an AC circuit the voltage across a resistive component is given by

$V(t) = 10 + 5\sin(3t)$

Find (a) The voltage amplitude

(b) Root mean square voltage across the component V_{rms}.

(The frequencies and hence phase differences between two components in an AC circuit varies with time so we can't use superposition methods here, we have to go with basics)

This waveform is the combination of one DC component of 10 V and the AC component 5 sin 3t which have a peak value of 5 V.

The DC component remains constant. Thus, the peak value of the waveform will be

$V_{max} = 10 + 5 = \mathbf{15\ V}$

For RMS value, we derive the average power dissipated in a resistor R.

The average power dissipated in the resistor for one time period is given by

$$P_{av} = \frac{1}{T}\int \frac{V^2}{R}\,dt$$

Or $$P_{av} = \frac{1}{R\,T}\int_0^T (10 + 5\sin 3t)^2\,dt$$

Or $$P_{av} = \frac{1}{R\,T}\int_0^T (100 + 100\sin 3t + 25\,sin^2\,3t)\,dt$$

Or $$P_{av} = \frac{1}{R\,T}\int_0^T \left(100 + 100\sin 3t + \frac{25}{2}\,(1 - \cos 6t)\right)\,dt$$

Or $$P_{av} = \frac{1}{R\,T}\left[100t - \frac{100}{3}\cos 3t + \frac{25}{2}\left(t - \frac{\sin 6t}{6}\right)\right]_0^T$$

Or $$P_{av} = \frac{1}{R\,T}\left[100\,T - \frac{100}{3}(\cos 3T - \cos 0^o) + \frac{25}{2}T - \left(\frac{\sin 6T}{6} - 0\right)\right]$$

Now here $\omega = 3$ and hence $T = 2\pi/3$;

Substituting the value of T in trigonometric functions we get

$$P_{av} = \frac{1}{R\,T}\left[100\,T - \frac{100}{3}(\cos 2\pi - \cos 0^o) + \frac{25}{2}T - \left(\frac{\sin 4\pi}{6} - 0\right)\right]$$

Or $$P_{av} = \frac{1}{R\,T}\left[100\,T + \frac{25}{2}T\right]$$

Or $$P_{av} = \frac{1}{R}\left[100 + \frac{25}{2}\right] = \frac{1}{R}[112.5]$$

As we know that the average power in the AC circuit is given by

$$P_{av} = \frac{V_{rms}^2}{R}$$

Thus $$V_{rms}^2 = 112.5$$

Or $$V_{rms} = \mathbf{10.61\ V}$$