(a) A space explorer comes across a spherical planet with a small moon in orbit around it. He observes that the moon completes one full orbit every 33 Earth days and moves at a distance *R* = 390 million m from the center of the planet. What is the mass of the planet? (One earth day =86400 s)

Let the mass of the planet be M and that of the moon is m. The gravitational pull of the planet is providing the necessary centripetal force to the moon and hence we can write

$$\frac{GMm}{R^2} = m\omega^2 R$$ ω is the angular velocity of the moon

Or $$\frac{GM}{R^2} = \left(\frac{2\pi}{T}\right)^2 R$$ where T is time period of the moon's rotation

Or $$M = \left(\frac{2\pi}{T}\right)^2 \frac{R^3}{G} = \left(\frac{2*3.14}{33*86400}\right)^2 \frac{(390*10^6)^3}{6.67*10^{-11}} = 4.851*10^{-12}*8.893*10^{35}$$

Or M = $4.314*10^{24}$ kg

Answer: **M_{planet} =** $4.314*10^{24}$ **kg**

(b) Upon further exploration he finds another satellite orbiting the planet in one third the time the moon did. What is the orbital radius of that satellite?

If the time period is one third means it is three times as fast as the moon or the angular velocity of the satellite ω' is three times that of the planet (3ω) hence if the orbiting radius of the satellite is R' then we have

$$\frac{GMm}{R^2} = m\omega^2 R$$ for the moon and

$$\frac{GMm'}{R'^2} = m'\omega'^2 R'$$ for the satellite

Solving the two equations we have

$$GM = \omega^2 R^3 = \omega'^2 R'^3$$

gives $$\omega^2 R^3 = 9\omega^2 R'^3$$

or $$R' = \left(\frac{1}{9}\right)^{\frac{1}{3}} R = 0.4807*R = 0.4807*390*10^6 = 187.5*10^6 \text{ m}$$

Answer: **$S_{atellite}$ = 187.5 million m**

(c) The explorer now lands his spacecraft on the surface of the planet near the North Pole. He weighs himself and finds he weighs only $\boldsymbol{W_{planet}}$ **= 152 lbs** while on earth he weighed $\boldsymbol{W_{earth}}$ **= 182 lbs**. Assuming the explorer hasn't dieted on his trip, what is the radius of the planet?

The acceleration due to gravity at the surface of the planet is given by

$$g_{planet} = \frac{GM}{r^2} = \frac{6.67*10^{-11}*4.315*10^{24}}{r^2} = \frac{2.878*10^{14}}{r^2}$$

Here r is the radius of the planet.

If the mass of the person is m then

$$\frac{W_{earth}}{W_{planet}} = \frac{m*g_{earth}}{m*g_{Planet}} = \frac{g_{earth}}{g_{Planet}}$$

Hence we get

$$\frac{182}{152} = \frac{9.8*r^2}{2.878*10^{14}}$$

gives $r^2 = \frac{182*2.878*10^{14}}{152*9.8} = 3.5*10^{13}$

or r = 5929884.8 m = 5930 km

Answer: $\mathbf{R_{planet}}$ **= 5930 km**

(d) The explorer continues his studies and soon ends up at the equator. There he finds the scale reads **95%** of what it did at the North Pole. How long does it take the planet to make one complete rotation?

The weight of the man at the pole is

$W_{pole} = \frac{GMm}{r^2}$ -------- (1)

The weight of the man will reduced due to the centrifugal reaction of the planet. The effective value weight at the equator is given by

$W_{equa} = W_{pole} - m\omega^2 r$

Or $W_{pole} - W_{equa} = m\omega^2 r$ ------- (2)

Now according to the question

$(W_{pole} - W_{equa})/ W_{pole} = 5/100$

Hence substituting from equations (1) and (2) we get

$$\frac{m\omega^2 r^3}{GMm} = \frac{5}{100}$$

or $$\omega^2 = \frac{5GM}{100r^3} = \frac{5*6.67*10^{-11}*4.314*10^{24}}{100*(5930000)^3} = 6.9*10^{-8}$$

or ω = 2.63*10^{-4} rad/s

Hence time period is

$$T = \frac{2\pi}{\omega} = \frac{2*3.14}{2.63*10^{-4}} =$$ 23908 s = 6.64 hours.

Answer: **$T_{rotation}$ = 6.64 hours**

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