Q- A compound gear system driving a large roller is shown below.

(a) Calculate the angular velocities of gears B, C and the product roller for when gear A has an angular velocity of 200 rpm.

(b) When power is applied to the gear A it is required that the design running speed (of 200 rpm at gear A) be reached in a period not longer than 10 seconds when no other loads are applied to the roller. Calculate the starting torque required in these conditions. Neglect the mass of the shaft connecting gear B with gear C. The number of teeth, masses and diameters of the gears are:

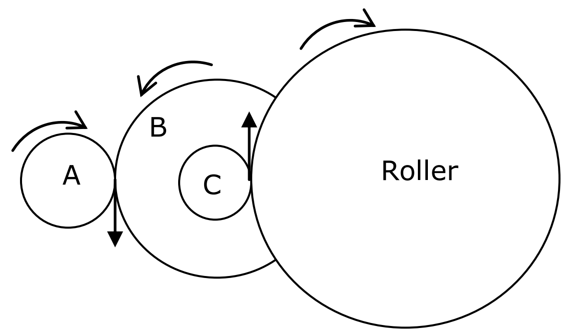
A: 50 tooth, 200 mm, 15 kg;

B: 200 tooth, 800 mm, 220 kg;

C: 70 tooth, 280 mm, 22 kg

The roller has an outer spur gear of 800 teeth and a diameter of 3200 mm. The mass of the roller is 20,000 kg.

a) When two wheels are geared and moves together, velocity of the point at the contact is same, as there is no slipping. The angular velocities are different in magnitude and decided by their radii. The directions of the angular velocities are opposite. The larger is the diameter the smaller is the angular velocity.



The angular velocity of A is ω_0 = 200 rpm = 200*2π/60 = 20π/3 = 20.944 radians/s.

Then angular velocity of B (and C as well) ω_1 is given by

$$\omega_1 R_B = -\omega_0 R_A$$

or $\omega_1 = -\omega_0*(R_A/R_B) = -\omega_0*(100/400) = -\omega_0/4 = -5.236$ radians/s

As B and C are fixed together the angular velocity of C is also $-\omega_0/4$ and hence angular velocity of D is ω_2 given by

$$\omega_2 = -\omega_1 (R_C/R_D) = (\omega_0/4)(140/1600) = 7\omega_0/320 = 0.458 \text{ radians/s.}$$

b) The moment of inertia of gears (considering disks) is

$$I_A = (1/2) M_A R_A^2 = 0.5*15*(0.1)^2 = 0.075 \text{ Kgm}^2$$

$$I_B = (1/2) M_B R_B^2 = 0.5*220*(0.4)^2 = 17.6 \text{ Kgm}^2$$

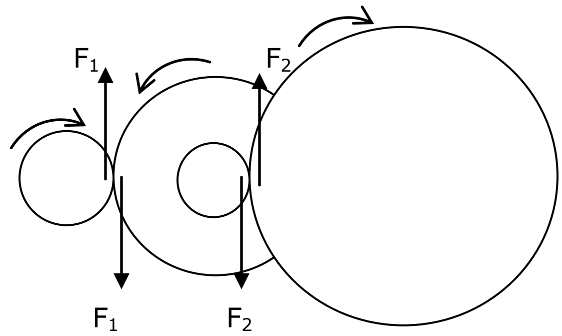
$I_C = (1/2) M_C R_C^2 = 0.5*22*(0.14)^2 = 0.2156 \text{ Kgm}^2$ and of the roller

$$I_D = (1/2) M_D R_D^2 = 0.5*20000*(1.60)^2 = 25600.0 \text{ Kgm}^2$$

Differentiating the relation of instantaneous angular velocity $\omega_1 = K\,\omega_2$ we get $\alpha_1 = K\,\alpha_2$ means that if the wheels or gears are attached in some way then the ratio of their angular acceleration is same as that of their angular velocities. Hence if α is the angular acceleration of the gear A then angular acceleration of B, C and D will be

$\alpha_B = \alpha_C = \alpha/4$

and $\alpha_D = 7\alpha/320$



Now as we start rotating A due to inertia B will try to stop it hence a pair of action and reaction produced between the tooths of A and B. let the magnitude of these forces be F_1 and similarly the force between C and D be F_2. The directions of these forces are indicated in the fig 2

Now if the external torque acting on A be τ then writing equations of rotational motion for the gears we have

For A $\tau - F_1R_A = I_A*\alpha$ ----------- (1)

For B&C $F_1R_B - F_2R_C = (I_B + I_C)*\alpha/4$ ----------- (2)

And for D $F_2R_D = I_D *7\alpha/320$ ----------- (3)

But the angular acceleration of A should be such that in 10 s it acquires an angular velocity $\omega_{0,}$ hence

Using equation of motion $\omega_0 = 0 + \alpha*t$

Gives $\alpha = \omega_0/t = 20.944/10 = 2.0944$ rad/s/s

Hence from equation 3 on substituting the values we have

$F_2*1.60 = 25600.0*(7/320)*2.0944$

Gives $F_2 = 733.04$ N

Substituting in equation2 we have

$F_1*0.4 - 733.04*0.14 = (17.6 +0.2156)*2.0944/4$

Gives $F_1 = 279.88$ N

And from equation 1 we have

$\tau = F_1R_A + I_A*\alpha$

or $\tau = 279.88*0.1 + 0.075*2.0944 = 280.04$ Nm

This is the required torque.

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