Q-3 The two problems below are related to a cart of mass M = 500 kg **going around a circular loop-the-loop of radius** R = 10 m**, as shown in the figures. All surfaces are frictionless. In order for the cart to negotiate the loop safely, the normal force exerted by the track on the cart at the top of the loop must be at least equal to** 0.4 **times the weight of the cart. You may neglect the size of the cart. (Note: This is different from the conditions needed to "just negotiate" the loop.)**

a) For this part, the cart slides down a frictionless track before encountering the loop. What is the minimum height **h** above the top of the loop that the cart can be released from rest in order that it safely negotiates the loop?

The normal force exerted will be minimum at the top of the loop the loop circle. If it it just 0.4 mg then total downward force at the top of the loop will be

$$mg + 0.4\ mg = 1.4\ mg$$

at the top this force is providing the necessary centripetal force and hence if the speed of the cart at the top is v, we have

$$mv^2/R = 1.4\ mg$$

or $$mv^2 = 1.4\ mgR$$

or $$v^2 = 1.4\ gR \qquad \text{---- (1)}$$

The loss in the height between the initial position and the top of the loop is h, hence according to law of conservation of energy the kinetic energy of the cart at the top is given by

$$m*g*h = ½\ mv^2$$

Substituting the value of v^2 from equation (1) we get

$$h = 0.7\ R = 0.7*10 = 7\ m$$

Answer: **h = 7.0 m**

b) For this part, we launch the cart horizontally along a surface at the same height as the bottom of the loop by releasing it from rest from a compressed spring with spring constant **k = 10000 N/m**. What is the minimum amount **X** that the spring must be compressed in order that the cart "safely" (as defined above) negotiate the loop?

As the normal force at the top is to be same, the velocity at the top and hence the kinetic energy should be the same.

Now in this case the energy supplied from the elastic potential energy of the spring. This energy should be sufficient to give the required gravitational potential energy and kinetic energy at the top. Hence

Loss of elastic PE = gain in gravitational PE + gain in KE

Or $$½\ Kx^2 = mg(2R) + ½\ mv^2$$

(the velocity v will be the same as in previous case) or $\frac{1}{2} Kx^2 = mg(2R) + \frac{1}{2} m(1.4gR)$

or $\frac{1}{2} Kx^2 = 2.7mgR$

or $x^2 = 5.4*mgR/K = 5.4*500*9.8*10/10000 = 26.46$

gives $x = 5.144$ m

Answer: X = 5.144 m

c) When the car is descending **vertically** (ie at a height **R** above the ground) in the loop, what is its speed **|v|**?

The velocity at height R, v_1 can be calculated using law of conservation of energy i.e.

Gain in KE = loss in PE

Or Final KE – initial KE = mg* loss in height

Or $\frac{1}{2} mv_1^2 - \frac{1}{2} mv^2 = mg*R$

Or $v_1^2 - v^2 = 2g*R$

Or $v_1^2 = v^2 + 2g*R = 1.4gR + 2gR = 3.4gR = 3.4*9.8*10 = 333.2$

Or $v_1 = 18.25$ m/s

Answer: **|v|** = 18.25 m/s

d) At the bottom of the loop, on the flat part of the track, the cart must be stopped in a distance of **d = 10 m**. What retarding acceleration **|a|** is required?
Velocity at the bottom of the loop v2 can be calculated in the similar method by conserving energy at the top and bottom points.

Gain in KE = loss in PE
Or Final KE – initial KE = mg* loss in height
Or $\frac{1}{2} mv_2^2 - \frac{1}{2} mv^2 = mg*2R$
Or $v_2^2 - v^2 = 4g*R$
Or $v_2^2 = v^2 + 4g*R = 1.4gR + 4gR = 5.4gR = 5.4*9.8*10 = 529.2$
Or $v_2 = 23.00$ m/s

To stop it in distance d the required declaration a is given by the third equation of motion as
$[v^2 = u^2 + 2as]$

$0 = 23^2 + 2a*10$
gives $a = -26.46$ m/s^2
(negative is for deceleration).

Answer: |a| = 26.46 m/s^2