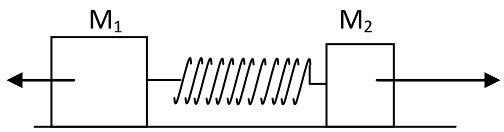
A massless spring of spring constant *20 N/m* is placed between two carts. Cart 1 has a mass $M_1 = 5\ kg$ and Cart 2 has a mass $M_2 = 3.5\ kg$. The carts are pushed toward one another until the spring is compressed a distance *1.4 m*. The carts are then released and the spring pushes them apart. After the carts are free of the spring, what are their speeds?



In such problems we may consider the two masses as a system. The forces with which the bodies are interacting are called internal forces. There may be some external force too. Here vertical forces are balanced and no external horizontal force is acting on the system when they are released and hence we can apply law of conservation of linear momentum.

The law of conservation of linear momentum is derived from Newton's second law of motion and says, "If no external force is acting on the system, its linear momentum remains conserved."

Hence as in our system there is no external force on the system (in horizontal direction) after release, initial momentum is equal to final momentum.

As initially the bodies are at rest, the initially the initial linear momentum of the system is zero or

$$\vec{P}_{initial} = 0$$

Let the final velocities of the carts are $\vec{v}_1$ and $\vec{v}_2$ respectively, final momentum of the system is given by

$$\vec{P}_{final} = M_1\vec{v}_1 + M_2\vec{v}_2$$

Applying law of conservation of momentum we can write

$$\vec{P}_{final} = \vec{P}_{initial}$$

Or $$M_1\vec{v}_1 + M_2\vec{v}_2 = 0$$

Or $$\vec{v}_2 = -\left(\frac{M_1}{M_2}\right)\vec{v}_1 \qquad \text{-------------------------- (1)}$$

The negative sign is showing that the velocity of cart 2 is in opposite direction to that of cart 1.

Now as the elastic forces are conservative, there is no loss of energy and the elastic potential energy stored in the compressed spring will convert to the kinetic energy of the two carts and hence applying law of conservation of mechanical energy we get

Gain in K.E. = loss in elastic P.E

Or $\frac{1}{2}M_1\vec{v}_1^2 + \frac{1}{2}M_2\vec{v}_2^2 = \frac{1}{2}\ K\Delta l^2$

As initial kinetic energy of the carts was zero.

Here K is the spring constant and ❑l is the compression in the spring. The last term is the elastic potential energy of the compressed spring.

Substituting the magnitude of v_2 (squared quantity is always scalar) from equation (1) we have

$$\frac{1}{2}M_1\vec{v}_1^2 + \frac{1}{2}M_2\left(\frac{M_1}{M_2}\right)^2\vec{v}_1^2 = \frac{1}{2}\ K\Delta l^2$$

Or $\frac{1}{2}M_1\vec{v}_1^2\left(1 + \frac{M_1}{M_2}\right) = \frac{1}{2}\ K\Delta l^2$

Or $\vec{v}_1^2 = \frac{M_2\ K\Delta l^2}{M_1(M_1+M_2)}$

Substituting numerical values we get

$$\vec{v}_1^2 = \frac{3.5*20*(1.4)^2}{5*(5+3.5)} = 3.29$$

or v_1 = 1.80 m/s

Substituting this in equation (1) again we get the speed of the second cart as

v_2 = - (5/3.5) v_1 = 1.43*1.80 = 2.57 m/s