Q- consider an object of mass m, moving in a circular orbit, subject to a central attractive Force F, whose magnitude is given by F(r) = h/r^3.
(a) what are the dimensions of h?
(b) show that the angular momentum for the motion is uniquely determined by h and m.
(c) what is the resulting relation between period and radius analogous to Kepler's third law for this force?

Answer:

(a) The force F(r) is given by

$F = h/r^3$

Or $h = F*r^3$

Hence the product of dimensions of F and r^3 gives the dimensions of h

Or $$[h] = [MLT^{-2}]*[L]^3 = ML^4T^{-2}$$

(b) Let the mass of the object be m and speed of the object is v in the orbit of radius r. As we know that the centripetal force required moving the object in a circular path is given by mv^2/r and the central force provides this, we get

$$F(r) = \frac{h}{r^3} = \frac{mv^2}{r}$$

This gives

$$h = \frac{mv^2r^3}{r} = mv^2r^2$$

gives $$v*r = \sqrt{\frac{h}{m}} \quad \text{----------------------------- (1)}$$

Now as we know that the angular momentum of an object is the moment of momentum and can be given by m*v*r the angular momentum of the object using equation (1) is given by

$$L = m*v*r = m*\sqrt{\frac{h}{m}} = \sqrt{m*h}$$

Hence the angular momentum of the object is in terms of mass m and the constant h only.

(c) The time period of the object is given by

$T = 2\pi/\omega = 2\pi r/v$

Where ω is the angular velocity of the object and is equal to v/r.

Substituting the value of v from equation (1) we get

$$T = 2\pi r(1/v) = 2\pi r*r\sqrt{\frac{m}{h}}$$

As m and h are constants, we have the relation

$$T \alpha r^2$$

Or the time period of the object is proportional to the square of the distance from the center of circular path.