Q- A string vibrates at its third harmonic frequency. The amplitude at a point 30 cm from one end is half the maximum amplitude. How long is the string?

The equation of standing wave on stretched string is given by (x=0 at one end)

$$y(x,t) = A \sin(kx)\cos(\omega t)$$

Here k is the wave number $2\pi/\lambda$ and ω is the angular frequency.

And thus, the amplitude at a distance x from the end is given by

$$A_x = A\ sin\ (kx)$$

Now as the amplitude Ax is half the maximum amplitude A at distance x = 30 cm we get

$$\frac{A}{2} = A\ sin\ (kx)$$

Gives $sin(kx) = \frac{1}{2}$

Or $kx = \frac{\pi}{6}$

Substituting the value of k and x we get

$$\frac{2\pi}{\lambda} * 30 = \frac{\pi}{6}$$

Or λ = 360 cm

Now as the string is vibrating in its third harmonic, there will be three loops of length $\lambda/2$ and hence total length of the string will be

L = 3*(λ/2) = 3*360/2 = 540 cm = **5.40 m**