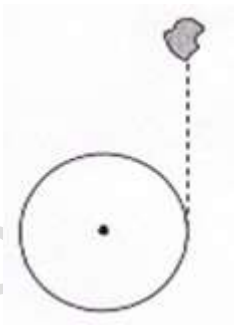
Q- A solid disk of mass M = 1.3 kg and radius R = 25 cm is free to rotate in a vertical plane about a horizontal axis going through its centre. The disk is originally at rest. A piece of clay of mass m= 200 gm (consider particle) drops from 2.5 m above the centre of the disk and sticks to its edge as shown. Find the angular speed of the disk just after the collision.



Moment of inertia of the disk about its axis of rotation

$I = \frac{1}{2} MR^2$

Velocity of the clay just before collision

$$v = \sqrt{2gh} = \sqrt{2 * 9.8 * 2.5} = 7\ m/s$$

Momentum of the clay just before the collision is m v

Now when a particle moves in a straight line it posses angular momentum about an axis passing through a point not in the line of motion. That is why angular momentum is called moment of momentum. The measure of angular momentum of a particle about an axis is given by the product of momentum of the particle and the perpendicular distance of the line of motion of the particle from the axis of rotation. In Vector form it is given as

$$\vec{L} = \vec{r} \times \vec{P}$$

Hence angular momentum of the particle (clay) about the axis of rotation of the disk is given by

$L = P*R = mv\ R$

When the clay sticks to the disk their total moment of inertia will be

$I' = I + mR^2$

Now if we consider the clay and the disk as a rotating system, just at the time of impact no external torque is acting on the system (as the time of impact is negligible the impulsive torque of the weight of the clay about the axis can be considered zero) according to law of conservation of angular momentum we get

Angular momentum of the system before impact = final angular momentum

or $I*0 + mv*R = (I + mR^2)\omega$

where w is the angular viscosity of the disk after collision with the clay

gives $\omega = \frac{mvR}{I+mR^2} = \frac{mvR}{(1/2)MR^2+mR^2}$

or $\omega = \frac{2mv}{(M+2m)R}$

gives $\omega = \frac{2*0.2*7}{(1.3+2*0.2)*0.25} = 6.59\ rad/s$