

Q - An AC generator supplies an rms voltage of 5.00 V to an RC circuit. At a frequency of 20.0 kHz the rms current in the circuit is 37.0 mA; at a frequency of 30.0 kHz the rms current is 50.0 mA. What are the values of R and C in this circuit?

Answer:

The impedance of a RC circuit is given by $Z = \sqrt{R^2 + (1/C\omega)^2}$ which depends on $\omega = 2\pi n$.

Now $\omega_1 = 2\pi n_1 = 2\pi \times 20000 = 125660 \text{ rad/s}$

and $\omega_2 = 2\pi n_2 = 2\pi \times 30000 = 188400 \text{ rad/s}$

The rms current is given by $I = V/Z$ hence $Z = V/I$ gives

$$R + (1/C\omega)^2 = V/I$$

For first case $Z_1^2 = R^2 + (1/C\omega_1)^2 = (5 / 37 \times 10^{-3})^2 = 18260 \dots \dots \dots (1)$

and for second $Z_2^2 = R^2 + (1/C\omega_2)^2 = (5 / 50 \times 10^{-3})^2 = 10000 \dots \dots \dots (2)$

Subtracting the two equations

$$(1/C)^2 * (1/\omega_1^2 - 1/\omega_2^2) = 8260$$

Or $(1/C)^2 * (1/4\pi^2 n_1^2 - 1/4\pi n_2^2) = 8260$

Or $(1/2\pi C)^2 * (1/n_1^2 - 1/n_2^2) = 8260$

Or $(1/2\pi C)^2 * 1.3889 * 10^{-9} = 8260$

Or $(1/2\pi C)^2 = (8260/1.3889) * 10^9 = 5.95 * 10^{12}$

Or $2\pi C = 4.098 * 10^{-7}$

Gives $C = 6.52 \times 10^{-8} \text{ F} = \mathbf{65.2 \text{ nF}}$

Substituting in eq (2) we get

$$R^2 = 10000 - (1/C\omega_2)^2$$

Or $R^2 = 10000 - (1/65.2 * 10^{-9} * 188400)^2$

Or $R^2 = 10000 - 6627 = 3373$

Gives $R = \mathbf{58 \Omega}$