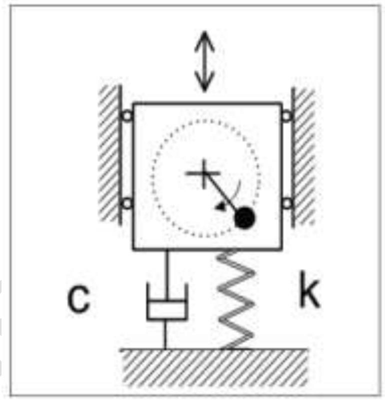
Q- A simple fatigue testing machine has a moving base that is 15kg and is constrained to only move vertically. The base is supported by a rubber block with a spring constant, k = 18 kN/m and a viscous damping constant of c = 200N.s/m. A rotating mass which is part of the base creates a harmonic force of 60N.

a) If the mass is rotated at 1500rpm, calculate the amplitude of vibration of the base

b) At what speed is maximum amplitude attained and what is the amplitude at this speed?



This is the case of the forced damped harmonic oscillator.

The rotating mass requires a centripetal force whose reaction is acting on the base and it forced vibration in the base. As the base is constrained to move in vertical direction the component of the reaction force in vertical direction is given by $F_0 * \cos \omega t$, Where ω is the angular speed of the mass

The net force on the base is given by the resultant of the spring force, the drag force and the reaction of the centripetal force of the mass. (The weight is balanced by the tension in the spring due to initial compression in the spring to come to equilibrium position and here y is measured from the equilibrium position). Hence the net force on the base is given by

$$F = -K y - c v + F_0 \cos \omega t$$

The equation may be written as

$$m\ddot{y} = -Ky - c\dot{y} + F_0 cos\ \omega t$$

Or $$m\ddot{y} + Ky + c\dot{y} = F_0 cos\ \omega t$$

Or $$\ddot{y} + \frac{c}{m}\dot{y} + \frac{K}{m}y = \frac{F_0}{m} cos\ \omega t$$

Or $$\ddot{y} + \gamma\dot{y} + \omega_0^2 y = \frac{F_0}{m} cos\ \omega t$$

Here γ is c/m and ω_0 is the natural frequency of the base with the spring only.

The solution of this equation is of the form

$$y = A\cos(\omega t + \emptyset)$$

Where the amplitude A is given by

$$A = \frac{F_0}{m} * \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\gamma)^2}} \quad \text{------------------- (1)}$$

And the phase difference ϕ is given by

$$\emptyset = tan^{-1}\left(\frac{\omega\gamma}{\omega^2 - \omega_0^2}\right)$$

Now in our question

F_0 = 60 N

m = 15 kg

γ = c/m = 200/15 = 13.33

$$\omega_0^2 = \frac{K}{m} = \frac{18000}{15} = 1200$$

And $\omega = 1500\, rpm = \frac{1500*2*3.14}{60} = 157\, radians/s$

(a) Substituting value in equation (1) we get

$$A = \frac{60}{15} * \frac{1}{\sqrt{(1200 - 157^2)^2 + (157 * 13.33)^2}} = 1.70 * 10^{-4}\, m$$

(b) The maximum amplitude will be attained when the denominator is minimum or when

$$\omega_0^2 - \omega^2 = 0$$

Or $\omega^2 = \omega_0^2 = 1200$

Or ω = 34.64 radians/s = 34.64*60/(2π) = 331 rpm.

And the maximum amplitude will be

$$A_{max} = \frac{F_0}{m} * \frac{1}{\omega_0\, \gamma} = \frac{60}{15} * \frac{1}{34.64 * 13.33} = 8.66 * 10^{-3}\, m$$

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ref: An Introduction to Mechanics (Danial Kleppner and Robert J Kolenkow)
McGraw-Hill International Editions